QUANTITATIVE DETERMINANTS OF CAPITAL INVESTMENT SELECTION

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A Thesis Submitted to the School of Government and Business Administration of The George Washington University in Partial Fulfillment of the Requirements for the Degree of Master of Business Administration

April, 1971

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INTRODUCTION

Subject

The selection of capital projects is one of the most important and critical business decisions and, as such, should be the preserve of top management.[1] For management is originally hired to control and direct the stockholders' (residual owners') funds and to maximize their earning power.[2]

Investment decisions are strategic in that they form the framework for a company's future development. Being a major determinant of efficiency and competitive power, they may well determine the success or failure of the enterprise.[3] What is more, the ability or inability of industry to plan for the orderly and timely replacement and expansion of facilities and equipment can have significant impact upon the country as a whole and quite properly become a concern of national policy.

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[1] A. J. Merrett & Allen Sykes, *The Finance and Analysis of Capital Projects* (New York: John Wiley & Sons, Inc., 1963), p. xi.

[2] Joel Dean, *Capital Budgeting* (New York: Columbia University Press, 1951), p. 1.

[3] *Financial Handbook*, 4th ed., edited by Jules I. Bogen (New York: The Ronald Press Company, 1964), p. 17.4.

> *No country can contemplate with equanimity the failure of its industry to keep abreast of technology. It deprives the state of power and security and robs the citizens of the advance in living standards to which he is properly entitled. When private enterprise dedevelops a predilection for antinques as instruments of production it can expect sooner or later to come under the critical scrutiny of the state.*[1]

But despite the widely acclaimed importance of sound investment policy and the availability of concepts and methods to help management in this area, the procedures frequently used in investment decisions are almost unbelievably primitive.[2] The managements that have thought through the basic conceptual, or analytical, problems of investment policy are relatively few.[3] As a result of this shortcoming, many companies are unaware of their investment prospects, or deliberately pass them up for lack of funds. A compendium of this problem is the situation of England, as poignantly portrayed by Merrett and Sykes.

> *Whereas individual. . . industries may excape. . . with an inadequate rate of growth, an individual nation is not in this position. Of no important Western nation is this more true than Britain where. . . inadequate growth has reached significant proportions. While. . . commonly thought to be due to an inadequate amount of investment. . . it is rather the quality of investment which is at fault.*[4]

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[1] George Terborgh, *Business Investment Management: An MAPI study and Manual* (Wash., D.C.: Machinery and Allied Products Institute, 1967) p. xxiii.

[2] Harold Biermand, Jr., and Seymour Smidt, *The Capital Budgeting Decision* (New York: The McMillan Company, 1960), p. 2.

[3] Terborgh, *op. cit.*, p. xxii.

[4] Merrett & Sykes, *op. cit.*, p. xii.

Terborgh has suggested a panacean scenario at the corporate level utilizing an adequate staff to search for investment opportunities, screen alternative solutions to each investment problem, perform adequate benefit analysis, and using correct justification tests, finally rank and recommend selection of projects in an orderly manner. He also cites the need for auditing the results of decisions, and for intelligent financial policy, inclusive of a rational investment threshold or cutoff point. [1] This comprehensive viewpoint is, in essence, a precept for good capital budgeting, an endeavor practiced with varying degrees of skill, magnitude and consistency by most large manufacturing firms.

Sound capital budgeting means good capital expenditure management, the very heart of which is the measurement of the investment worth of individual proposals.[2] It also entails the use of defensible objective standards for accepting an investment and an understanding of the economic content of the concepts used.

As has been suggested by some authoritative sources, a given investment measurement tool may not have the same connotation to all firms. Transcending the method is the way in which it is used. The choice of a method is apparently heavily influenced by the familiarity of management, or the capital budgeting staff, with the application of the procedure. This does not assure, however, that the underlying principles of the method are thoroughly

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[1]Terborgh, *op. cit.* p. xx.

[2]Joel Dean, "*Measuring the Productivity of Capital,*" *Harvard Business Review, 23* (January-February, 1954), pp. 120-129, passim.

understood. Consequently, the answers sought by management may be inconsistent with the rigid mathematical framework (and limitations) of the measurement apparatus management is using. Thus, given the variety of conditions that may face a firm selecting among investment proposals, and assuming an adequate yardstick for measuring investment worth has been selected, the next concern would gravitate around the use of the yardstick.[1] That is required in the way of inputs, and under what assumptions and frames of reference should these inputs be acquired and incorporated?

While the problem of acquiring, assembling and relating quantifiable inputs for the investment decision can be an onerous task, the real difficulty is in the assumptions and their impact. Each assumption involves its own degree of uncertainty; and, taken together, these combined uncertainties can produce an uncertainty factor of such significance that it cannot be rationally ignored. Some viable means of risk measurement would seem a necessary consideration in the capital investment decision.[2]

Having selected a means of measuring capital investment worth, and having grasped the intrinsic parameters and variables associated with the selected method, the rational manager would seek a standard against which to measure the resultant numerical

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[1]Dean, *op. cit., passim.*

[2]David B. Hertz, *Risk Analysis in Capital Investment, Harvard Business Review* (January-February 1964), p. 95.

expression. Herein lies one of the greatest controversies in capital budgeting. What standard, and what should it represent? While excursions into the labyrinth of theories of capital cost and structure may be beyond the job-scope of some capital budgeting staff positions, an appreciation of the relationship between cost of capital and the investment selection process might serve to sharpen the financial manager's horizon and enhance his piloting of the firm's investment program.

The Research Question

The principal question of this paper is: What primary quantitative factors govern selection of capital investments? Subsidiary to the basic question are:

A. What methodology is best suited to the measurement of capital investment worth?

B. Of what significance is the reinvestment assumption to capital investment selection?

C. What conceptual considerations comprise an investment cash flow analysis?

D. How should risk and uncertainty be applied to the investment selection process?

E. What is the relationship among investment selection, cost of capital and capital structure?

Limitations

This paper is primarily concerned with the quantifiable aspects of capital investment selection. It is recognized that a specific investment selection may be influenced by financing considerations and that often there is a leasing alternative. However, it is assumed, unless otherwise stated, that decisions on financing have been made and remain constant. The leasing alternative is considered beyond the scope of this paper.

Also appreciated are the organizational and administrative aspects and managerial philosophies that often play a large part in the company's capital budgeting effectiveness, and fashion the environment in which the investment decision is evaluated. These considerations will be interjected only where they complement the central theme.

The subject matter is developed with the large established industrial firm in mind, wherein the investment program is varied and the nature of the individual investment opportunity is complex. However, the material supporting the research would apply generally to any business where continuing investment is essential to survival.

The study is oriented to the investment policy-making and decision prerogatives of top management and the staff tasked with investment analysis. The interests of stockholders and the viewpoints of prospective investors are included where such subjective considerations enhance the discussion at hand.

Methodology

The intention of this writer was to survey, by library research, the multitudinous and often conflicting aspects of investment selection, and by integration, apposition and differentiation, derive a coherent, quantitative framework for selection of capital investments.

An initial problem in the organization and documentation of material for this paper was the seemingly endless array of writings on capital budgeting. Many authors, in marketing somewhat polemical views, seem to blur the distinction between hypothesis and principle. Another source of confusion was the multi-interpretive nature of some of the basic terminology.

Researching the selected bibliographies of a handful of widely acclaimed authorities on capital budgeting provided a formidable and fairly homogeneous reading base. To then extract an in-depth coverage on the narrow band of capital budgeting planned for the paper, a matrix of tables of contents helped segment the reading base by relevance and priority.

Organization

Chapter II provides a descriptive and evaluative cross section of significant investment worth measurement techniques available to management. From the contexture of investment selection, the mechanical and conceptual aspects of the various methods are introduced and examined. Of particular significance is the evaluation of the techniques with

regard to the time-value-of-money concept. Pursuit of this objective leads to the comparative analysis of two discounted cash-flow methods. Innovative methods of recent origin are included as a sequel to the basic theme of the chapter.

Factors essential to the use of worth measurement tools are treated normatively and descriptively in Chapter III. Procedural theory is followed by discussion of associated concepts such as differentiation of investment returns, timing, and the implications of taxation.

Chapter IV analyzes risk and uncertainty, the inevitable companions of the cash-flow process. Risk identification is followed by methods of risk quantification, and application to improved cash-flow forecasts.

In Capter V, some conventional models are presented to illustrate the nature of a weighted cost of capital. This composite rate is then put forth as a postulate in investment selection.

Chapter VI summarizes the findings of the study and concludes the paper.

CHAPTER II

METHODS OF INVESTMENT WORTH MEASUREMENT

In the process of weighing alternative investments, it is essential that the various proposals be evaluated as nearly as possible on a uniform, comparable basis. Quantitative methods provide a means for organizing the normally complex mass of data inherent in the investment evaluation process. When applied in proper perspective, they provide an explicit measure of relative worth of investment opportunities, obviating considerable guesswork from management decision-making.[1] While a measure of the economic worth of an investment normally cannot be the exclusive evaluative factor in the final decision, it should play an important part in the majority of investment proposals considered by the firm.[2] Logically, the measure of investment worth that leads to a maximization of profits is a valuable aid in investment decision-making.[3]

Judgment of the total selective process of investment proposals is involved in the selection of data used in the analysis, (such as revenue forecasts), in the estimation of project life, in subjective probability determinations and in

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[1]Robert G. Murdick and Donald D. Deming, *The Management of Capital Expenditures*, (New York: McGraw-Hill Book Co., 1968), p. 63.

[2]Bierman and Smidt, *op. cit.*, p. 11.

[3]*Ibid.*

the final decision itself.[1]

The analyst must be cognizant of certain implicit assumptions which may or may not be evident in the analysis scenario. For example, the cost/benefit comparison of two types of conveyor systems might assume:

1. Different load capacities at different costs.
2. Identical maximum speeds but different speed controls at:
   a. the same costs;
   b. different costs.
3. Different lives and operational costs.

Thus, a method of analysis which compares *only* absolute costs or cost savings implies that all other factors remain the same.[2] If this is not the case, the analyst must first establish trade-offs in terms of the variables being compared.

Thus, the method requires the user to apply judgment--to process an understanding, if only rudimentary, of the principles and assumptions underlying the applications.

Methods of Analysis Indifferent to Cash Flow Timing

Payback Method

The payback method is a simple, easily used and widely practiced quantitative method of evaluating proposals. It gives

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[1]Murdick and Deming, *op. cit.*, p. 63-64.

[2]*Ibid.* p. 64.

the number of years required for cash benefits to pay for the cost of an investment.

$$\text{Payback period} = \frac{\text{total original investment}}{\text{average cash benefits per year}}$$

or

$$= \frac{\text{total original investment (net)}}{\text{average profits after taxes and depreciation}}$$

For example, if $100,000 is to be invested in equipment, $20,000 is required for working capital, the old equipment has a salvage value of $5,000, profits after taxes are projected at $15,000 per year and depreciation charges will be $12,000 per year, the after-tax payback time will be:[1]

$$\text{Payback period} = \frac{100,000 + 20,000 - 5,000}{15,000 + 12,000} = 4.3 \text{ years}$$

It is surprising that approximately one-third of the largest industrial corporations in the United States used this method as the sole approach for determining rate of return, and half use it either alone or in combination with other methods,[2] for it has three striking disadvantages:

1. *It neglects the time value of money and capital additions. For instance, in the above example, it gives the same weight to the $15,000 in year four as that in year one.*

2. *No return on investment is derived to compare with the cost of capital (or an assumed cutoff rate). The resultant time required for investment recovery*

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[1]Assumes an investment credit against income tax is not in effect.

[2]Bogen, *op. cit.*, p. 22.

is useful for comparison with some arbitrary time period set by management--normally the economic life of the asset--and little else.[1]

3. *There is no consideration given revenue beyond the payback period.*[2] *(Assuming the project in the example generated a per annum return of $15,000 for five years, it would be equated with indifference to another undertaking with the same initial outlay but a return of $15,000 for eight years.*

For certain types of investments, such as those in politically unstable areas, payback period can be used to indicate relative risk and, hence, provide management with an additional influence factor.

George Terborgh states that as a device, the payback period does not test profitability and is seldom a test of capital recovery. He concedes that it does create a *presumption of relative attractiveness* for projects of similar characteristics. He further classifies it as *crude and fallible* and states that intelligent management should seek something better.[3] Relatedly, *Return on Investment* is equivalent to the reciprocal of the payback formula.

$$\text{Return on investment} = \frac{\text{average annual cash benefits}}{\text{total original investment (net)}}$$

or

$$\text{Cash-flow return after taxes} = \frac{\text{averaged profits after taxes and depreciation}}{\text{total investment (net)}}$$

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[1]Bogen, *op. cit.*, p. 37.

[2]James T. S. Porterfield, *Investment Decisions and Capital Costs* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), p. 21.

[3]Terborgh, *op. cit.*, p. 103.

Some accountants use gross investment rather than the more objective net investment for replacement analysis. The latter reflects salvage value and may or may not include removal costs. Also, the accounting practice of using net income in the numerator reduces the apparent return, as indicated below.[1]

$$\text{Return on investment} = \frac{\text{profits after taxes (averaged)}}{\text{gross investment}}$$

As in the payback method, no adjustment is made for the time value of money.

<u>The Accounting or Average Rate of Return Method</u>

This method is used by roughly 15 percent of the 500 largest industrial corporations as their only approach, and over one-quarter of them use it in combination with other methods.[2] In this method, the average annual earnings over the life of the investment are compared with either the average or the initial investment. There is considerable variety in the interpretation of "average earnings" and "investment by firms using this method. Although this technique gives a rate of return as a measure of performance, it does not take into account the timing of money flows. It, therefore, provides the same rate of return for projects with the same average earnings and investment, though one project may give a much more rapid rate of repayment than the other.[3] Anthony calls it the

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[1]Murdick and Deming, *op. cit.*, p. 68.
[2]Bogen, *op. cit.*, p. 22.
[3]*Ibid.*, p. 23

"unadjusted return on investment" and states that when calculated on the initial investment, it will always understate the true rate of return as found under the discounted cash flow methods.[1] It ignores the time value of money and assumes profits are fairly constant.

An example of this method assumes an investment of $9,000 for a machine, a requirement of $5,000 working capital, and an expected after-tax profit averaging $2,000 per year over the five-year life of the asset.[2]

Average investment = $5,000 + 1/2 (9,000)

$$\text{Average rate of return} = \frac{\text{average annual net income after taxes}}{\text{average investment over the project life}}$$

$$= \frac{2{,}000}{9{,}500} = 21\%$$

It is to be noted that had the initial investment been used instead of the average investment, the rate of return would have been approximately 14 percent (2,000 ÷ 14,000). Other variations are possible by using only the investment in the fixed asset, ignoring the working capital.

Time Adjusted Methods

Several methods for measuring the value of an investment have been considered. None succeeded in bringing the timing of cash flows into the analysis and some failed to consider the investment life. The discussion will now turn to

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[1] Robert N. Anthony, *Management Accounting: Text and Cases* (4th ed.; Homewood, Illinois: Richard D. Irwin, Inc., 1970), p.632.

[2] Murdick and Deming, *op. cit.*, p. 70.

measures of investment worth that employ means to evaluate the timing of the investment. As a group, these are known as discounted cash flow measures. However, before presenting them, an explanation is considered appropriate as to the nature of interest and the concept of present value of future amounts--both of which are utilized in the discounted cash flow measures.

A dollar received today can be worth more than a dollar received some time in the future. There is an alleged psychological phenomenon known as the *time preference* whereby people will refrain from consumption and loan the unspent money for the interest it will earn. However, disliking postponement of consumption, these people will only be induced to delay spending by interest payments sufficient to compensate for the disutility of the delay. Normally, the higher the interest rate, the more consumption they will be inclined to postpone.[1] Implied here is the fact that a roundabout capitalistic process will necessitate greater return than a less capital-intensive process. Time is money, and the longer process, utilizing resources for a longer period of time, must be commensurately more productive.[2]

Another reason for the greater value of the present dollar is the risk involved in the future dollar. A sum of money promised at some future date is by no means a certainty--

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[1] William J. Baumol, *Economic Theory and Operations Analysis* (2nd ed.; Englewood Cliffs, N.J.; Prentice-Hall, Inc.,1965), p.411.

[2] *Ibid.*, p. 415.

something could intervene. So, in addition to his disutility, the lender demands interest as compensation for the risk he incurs in making the loan.[1] Thus, for purposes of discussing time-adjusted investment-analysis methods, the rate of interest (or cost to the borrower) will be considered an expression of the lender's time preference and his risk consciousness. It is assumed that the borrower will seek a return on this loaned capital that will exceed its cost. Any delay in the process will entail added costs and, hence, require additional return.

The functional relationship of a future sum of money at time n to the present dollar is expressed as:

$$C = \sum_{i=1}^{i=n} \frac{Ai}{(1+r)^i}$$

where C = original investment (present value)

Ai = proceeds for a given year (cash flow)

r = rate of compound interest (discount rate)

i = year 1, 2 · · · n.

Here is the means of calculating the present value of any future cash flow. The total present value of a series of future cash flows is the algebraic sum of the present values of

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[1] *Ibid.*, 411. This point is discussed at length in Chapters IV and V.

the individual receipts and outlays. The expression n is the number of compound periods for the rate of compound interest and, therefore, represents discrete intervals of time.[1] Tables are available for continuous compounding as well as discrete compounding which greatly facilitate the use of this formula. The parenthetical terminology will be used and further explained in the ensuing discussion of time-adjusted methods.

The above formula is valid only when the effective interest or discount rate remain constant during the i to n period. If there are changes in rate, the present value must be computed in two or more steps. The future sum is discounted back at the respective interest rate to the time when the discount rate changes. Then, this value is discounted at the next rate, and so on.[2]

The Yield or Rate-of-Return Method

The yield method discussed here and the net present value method to follow are different approaches to the discounted cash flow technique. Both methods are based on the now familiar formula:

$$C = \sum_{i=1}^{i=n} \frac{Ai}{(1+r)^i}$$

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[1]If the compounding period is less than a year, for instance, every quarter, then the compound rate of interest, r , must be adjusted in the formula--in this instance to r/4.

[2]Porterfield, *op. cit.*, p. 24.

This equation is used in the yield method to solve for r. Where C is the initial cost of the investment, $A_2 \ldots An$ are cash flows in the years indicated by the subscripts, and r is the rate of return on investment, the object is to find r. (The distinction between this use of the basic formula and that of net present value is that in the above method no interest rate is assumed. In the net present value computation, r is the weighted cost of capital for the firm, and the equation is solved for C, the net present value.)[1]

In solving for r in the yield method, the financial analyst is faced with a series of terms. The equation is run through iterations using assumed values for r until the right-hand side (present value) equals the left-hand side (the investment outlay or cost). Raymond Reul proposed an alternate procedure whereby various discount rates would be charted against the resultant present values.[2]

Whatever the technique for solving the equation, the result is the maximum interest rate that can be paid for the capital employed over the life of the project (investment) without incurring a loss.[3]

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[1]Murdick and Deming, *op. cit.*, p. 73.

[2]Raymond I. Reul, "*Profitability Index for Investments*," *Harvard Business Review, XXXV, No. 4* (July-August, 1957), p. 118.

[3]"*Return on Capital as a Guide to Managerial Decisions*," National Association of Accountants, *Research Report #35* (New York: December, 1959), p. 57.

The discount rate that "equalizes" the two sides of the equation is the rate of return for the project which is directly comparable to the weighted cost of the capital used for the project (or a more subjective cutoff rate adopted by management). Whatever the standard or cutoff rate, the profitability of the prospective investment is judged by comparing the computed rate with the standard. For example, if the computed rate is 15 percent, and funds can be obtained at 10 percent, the investment is judged profitable. The higher the rate of return relative to the market cost of capital, the more profitable is the investment.[1]

This method can be used to rank projects and make "go/no-go" decisions; however, it has a limitation where the choice is between mutually exclusive projects. This matter will be explored after a discussion on the other time-adjusted method, net present value method.

The Net Present Value or Present Worth Method

Like the yield method, the net present value technique is one of the better methods of investment analysis because it not only takes into account the time value of money but also provides the flexibility to reflect depreciation and taxes as required when required. The present value concept entails discounting future costs and revenues in order to compare the present

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[1]Gerald A. Pollack, *"The Capital Budgeting Controversy: Present Value vs Discounted Cash-Flow Method," National Association of Accountants Bulletin,* (November, 1961), p. 2.

value of future benefits with the present value of the investment. If the net present value of the benefits does not exceed the investment, the investment should not be made.[1]

The mechanics of the process are as follows:[2]

1. Select an appropriate interest (discount) rate.

2. Calculate and estimate cash inflows for respective time intervals, which, as a minimum, include:

a. the tax effect (shield) of depreciation;
b. cash earnings, after tax;
c. residual asset value at the end of the economic life, inclusive of salvage value and/or disposition costs.

3. Calculate and estimate the cash outflows (to include startup costs and other initial outlays) as well as subsequent cash outlays for the year they are estimated to occur.

4. Find the net present value of all inflows and outflows by discounting them at the selected rate. The present value of the proceeds minus the present value of the outlays is the net present value of the investment.

The magnitude of the net present value serves as an indication of the relative worth of mutually exclusive projects. The recommended accept/reject criteria is zero. Thus, those independent projects with a present value of zero or above are considered acceptable for further evaluation (as may be prescribed by company policy) and those with a negative present value are normally dropped from further consideration.

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[1]Murdick and Deming, *op. cit.*, p. 71.

[2]Steps 1 through 3 also apply to the yield method, except that the interest rate selection is trial and error rather than a computed value such as the firm's cost of capital.

Since the present value of an investment will depend upon the rate of interest used, the present value will change if the interest rate changes. An investment may be feasible with one rate of interest, say, 10 percent, but unfeasible with a rate of 12 percent. Normally, a firm will either have a definite rate applied to all investments, or, less commonly, various rates depending on the investment classification.[1]

The present value of an investment, at the firm's derived cost of capital, may be described as the maximum amount the firm could pay for the opportunity of making the investment without experiencing a loss. Thus, the computed present value is a potential capital gain from an investment opportunity, over and above the minimum required return on the company's capital. This capital gain will materialize if the expected cash flows materialize.[2]

A ratio called the profitability index can be used to rank either mutually exclusive projects or projects that are not mutually exclusive but that do compete for available funds. The profitability index is obtained by dividing the present value of the cash inflows by the present value of the cash outflows.[3] An example of utilization of the index is

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[1]Adjustment of rates to compensate for risk is widely practiced but theoretically unsound. See *Return on Investment v. Cost of Capital* in Chapter V.

[2]Bierman and Smidt, *op. cit.*, p. 28.

[3]Anthony, *op. cit.*, p. 637.

illustrated below:[1]

A machine procurement has the following data:

Initial investment	\$18,000
Service life	10 years
Disposal value	\$ 5,000
S/L Depreciation	1,000/year
Interest rate	10%

Present value of the capital investment

$$= 18,000 \quad \frac{5,000}{(1 + 0.10)} \quad = 18,000 - 5,000\ (0.3855)$$

$$= 16,072$$

If V=present value (PV) of the cash flow [See Table 1] and C=PV of the investment,

$$\text{Profitability index} = \frac{V}{C} = \frac{23,390}{16,072} = 1.45$$

$$\text{Return on investment} = \frac{V-C}{C} = \frac{V}{C}-1 = 0.45 \text{ or } 45\%$$

If two projects with different service lives are being compared, they must be compared over the same period of time. The comparison will necessarily extend over multiples of the lives of each. Thus, if the service life of one project is three years and another is four years, the comparison period must be 12 years, with replacement occurring at the respective

[1]Murdick and Deming, *op. cit.*, pp. 71-72.

PRESENT VALUE METHOD OF DETERMINING INVESTMENT WORTH

End of year	Operating costs*	Revenue	Cash benefits	Cash benefits after taxes = 0.5* (CB+D)	Discount factor (10%)	Today's value
1	$12,000	$18,000	$ 6,000	$3,500	0.9091	$3,182
2	12,100	21,000	8,900	4,950	0.8264	4,091
3	12,200	22,000	9,800	5,400	0.7513	4,057
4	12,400	22,500	10,000	5,550	0.6830	3,791
5	12,600	21,000	8,400	4,700	0.6209	2,918
6	12,900	19,500	6,600	3,800	0.5645	2,145
7	13,300	18,000	4,700	2,850	0.5132	1,463
8	14,00	17,000	3,000	2,000	0.4665	933
9	15,000	16,000	1,000	1,000	0.4241	424
10	15,000	16,000	1,000	1,000	0.3855	386
	*Assumed values				Present value =	$23,390

Source: Murdick & Deming, *op. cit.*, pp. 71-72.

three- or four-year intervals. The present value of costs are compared (implying that both machines perform the same functions). Then the average cost per year over a comparable time period is computed in present-worth dollars for each project. Selection is based upon comparison of these average costs.[1]

Present Value and Yield Methods Compared

Of all the investment worth measurement procedures discussed to this point, neither the present value nor the yield methods could be dismissed as obviously incorrect.[2] If identical assumptions are made, (i.e., uniform rate of discount for all future cash flows and reinvestment of earnings at a certain definite rate), the results from the two concepts should be identical.[3] If a cash flow pattern of an investment is conventional (i.e., consists of one or more periods of cash outlays followed by periods of cash proceeds) and the cost of capital is used as a discount rate, the yield method will give the same "accept" or "reject" decisions as the present-value method. Inasmuch as this cash flow pattern is typical of a majority of investments, the generalization that, in practice, the yield and present-value methods give the same recommendations for *independent* investments is acceptable.[4] For instance, the yield

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[1] *Ibid.*, pp. 72-73

[2] Bierman and Smidt, *op. cit.*, p.34

[3] Robert W. Johnson, *Financial Management* (Boston: Allyn and Bacon, Inc., 1962), p. 176.

[4] Bierman and Smidt, *op. cit.*, p.35.

method will give the same results as the present-value method as long as the rate of discount at which it is appropriate to discount future cash flows is the same for all future years. Given this condition, the statement can be proven as follows: Under the present-value method:

$$V = \frac{A_1}{(1+i)} \quad \frac{A_2}{(1+i)^2} + \ldots \frac{An}{(1+i)^n}$$

and the borderline project is one where V=C, or V/C=1.

Under the yield method:

$$C = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \ldots \frac{An}{(1+r)^n}$$

and the marginal project is one which has a rate of return just equal to the market cost of capital, where r=i. But when V=C, it follows that r equal i.[1]

However, if the rate of interest varies from year to year, despite the fact that this pattern variation is correctly anticipated, the two procedures cannot be used in a way that will give identical answers. For example, the present-value formula can be applied when the market cost of capital is expected to change from year to year. The formula reads:

$$V = \frac{A_1}{(1+i_1)} + \frac{A_2}{(1+i_1)(1+i_2)} = \ldots \frac{An}{(1+i_1)(1+i_2)\ \ldots(1+i_n)}$$

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[1]Pollack, *op. cit.*, p. 10.

This refinement is not possible in the yield method, where a project rate of return is compared with the cost of capital, for there is no one market rate for a basis of comparison.[1] Thus, like most generalities, there are exceptions to the rule, and given the vagaries of financial practice in the business world, it would be unlikely that the two methods would consistently yield comparable results.[2] Differences in the results provided by the two methods under certain conditions are discussed under respective subsections farther on in this chapter.

The mechanical and conceptual aspects of the present-value method are less error-prone from a user standpoint than the more difficult calculations and complex procedural rules of the yield technique.

Merrett and Sykes plainly state preference for the yield method. However, after conceding the "decisive advantages over yield" of the present-value method "in certain special applications," their preference appears highly subjective, being stated that:[3]

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[1] *Ibid.*, p. 9.

[2] Reul, *op. cit.*, p. 117.

[3] Merrett and Sykes, *op. cit.*, pp. 148-150.

> *(1) Yield is a more useful measure of profitability when endeavoring to assess the return offered for risk-bearing--a rate of return per unit of capital outstanding in the project per unit of time is essentially measuring return in the same dimensions as risk (time and quantity...)*[1]
>
> *(2) Yield method, despite its being...more complicated...is more easily understood and accepted by businessmen....*
>
> *(3) The yield method has the advantage of obviating needless dispute about a firm's cost of capital.*[2]

The involved discussions that follow in the book on the adaptation of the yield method to offset its disadvantages appeared more as a defense of a cherished institution than as an objective evaluation. However, one assertion concerning subjective risk preference did project as realistic insight which is discussed later in the chapter.

The present-value approach has a tendency to reconcile the numerical significance of future cash flows with the decreasing reliability of future forecasts. That is, the more distant (and uncertain) the particular cash flow, the smaller the present value factor applied to the cash flow.[3] Of course, when investment costs are comprised almost exclusively of large initial outlays, this rough compensatory aspect would apply essentially to subsequent revenue and operating cost forecasts.

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[1]This line of reasoning conflicts with the *N.A.A. Research Report #35, op. cit.*, p. 64. Risk is discussed in Chapter IV.

[2]See *Cost of Capital*, Chapter V.

[3]*National Association of Accountants Bulletin, op. cit.*, p. 64.

This observation does not infer that the use of present-value discount factors will lead to a correct or appropriate allowance for uncertainty. In special situations where this technique is used to adjust for risk, the discounted cash flows should be referred to as expected cash flows adjusted for risk, vice present values; and present-value discount factors would then have to be applied to the estimates a second time.[1]

An advantage often attributed to the yield procedure is that it may be utilized without deciding on the cost of capital; whereas, the present-value method requires that the cost of capital be incorporated into the formula. While there is some transient advantage to being able to proceed with the ranking of competitive (non-mutually exclusive) projects, the argument falters in the face of the accept-or-reject type of investment decision. The yield of an investment must be compared with the cost of capital. Thus, the cost of capital is no less important to yield than to present value, although it enters at an earlier stage in the computation of the latter.[2]

There are a number of evaluative premises, or sets of circumstances, under which the conceptual nature of one method makes it superior to the other.

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[1] Bierman and Smidt, *op. cit.*, p. 55

[2] Bierman and Smidt, *op. cit.*, p. 36.

Mutually Exclusive Investments

Essentially the yield and present-value methods answer different questions. The yield method seeks to find the maximim rate of interest at which project capital outlays can be recaptured by project earnings. The present-value method asks what amount could be invested in a given project so that planned project earnings will equate to this amount, with interest at the market rate.[1]

It is not surprising then that under certain conditions the evaluate the same projects differently. In the absence of capital rationing[2] this behavior would not be so troublesome, as long as either properly indicated profitability. However, in many undertakings, a choice must be made among mutually exclusive projects, such as a factory site or a fleet of equipment. It is simply a case of one or another. But a measure of investment worth that does not lead to a correct choice can be a genuine liability. This can be demonstrated by a numerical example.[3]

> *Assume a choice is to be made between two projects, each requiring an investment of $1.00. The first returns nothing the first year, and $4.00 at the end of the second year. The second returns $2.00 at the end of the first year, and $1.00 at the end of the second.*

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[1] Pollack, *op. cit.*, p. 8.

[2] As used here, rationing applies only to the logical selection of a single mutually exclusive project rather than the usual connotation of limiting selections to a few prime candidates from a list of competitive projects.

[3] J. Hirshleifer, *"On the Theory of Optimal Investment Decision," The Journal of Political Economy*, August, 1958, 348.

These conditions are summarized as follows, with the minus sign indicating a cash outlay:

Time	0	1	2
Project I	-1	0	4
Project II	-1	2	1

Under the yield method, the rates of return of the two projects are:

Project I	=	100 percent
Project II	=	141.4 percent

However, under the present-value method (assuming a cost of capital of 10 percent), the ratios, V/C, of the two projects are:

Project I	=	3.31
Project II	=	2.64

The two methods obviously yield different answers to the same problem, a clearly unacceptable situation for a firm that can accept only one and wants the more profitable choice.

In examining the implications of the two methods, it can be conclusively demonstrated that the project with the higher present value and, consequently, the lower rate of return, is the superior investment.

Assuming an interest rate of 10 percent for Option I, and borrowing on the projected earnings of the final time period for the benefit of the intermediate one, -1, 0, 4 can be converted

to -1, 2.73, 1. (3 was subtracted from the final period, crediting the intermediate period with 3 ÷ 1.1, or 2.73).

Option II can be obtained by "losing" the 0.73, leaving -1, 2, 1. The fact that wealth has to be lost to get from Option I to Option II demonstrates the superiority of Option I even though the rate of return of Option II is greater than Option I. This example holds for interest rates up to 50 percent.[1]

By similar reasoning, it can be shown that whenever the two methods rank the same mutually exclusive projects in different order, the present-value method ranks them in an order which, assuming rational behavior on the investor's part, leads to higher profits[2]--the reason being that the yield method neglects incremental cash flows. This comparative advantage can best be illustrated with an example.

TABLE 2

COMPARISON OF INVESTMENT ALTERNATIVES WITH INCREMENTAL CASH FLOW BENEFITS

	Project A	Project B
Outlay	$10,000	$15,000
Proceeds (next year)	12,000	17,000
Yield	20%	18%

Source: Bierman and Smidt, *op. cit.*, p. 37.

[1]*Ibid.*, p. 348.

[2]Pollack, *op. cit.*, p. 11.

The difference in outlay for the two projects in Table 2 is $5,000 and the respective difference in proceeds is $5,700. The return on the incremental flow of $700 is 14 percent. Yet, taking the investments as a whole (ignoring the incremental cash flow) the smaller investment would be chosen. When only the yield of the entire investment is considered, something important is left out--the *size* of the investment.[1] Continuing to use the data in Table 2, the yield on a $10,000 investment is 20 percent; that of the alternative is 18 percent on $15,000. But if the cost of capital is less than 14 percent, the $10,000 investment is inferior despite its higher yield. Again, this assumes that a choice between the projects is mandatory.

This points up a disadvantage of the yield method. In order to determine which of a pair of mutually exclusive investments is preferable, it is necessary to compute the yield on the incremental cash flows. If there are more than two such investments in the running, they must be systematically evaluated and compared, pair by pair. The superior alternative of the first pair-off is comparedwith one of the remaining alternatives and this procedure repeated until the best of the lot is apparent by inspection.[2]

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[1]Bierman and Smidt, *op. cit.*, pp. 37-39.

[2]Bierman and Smidt, *op. cit.*, p. 40.

The Yield Method and Non-Mutually Exclusive Projects

Most authorities on capital budgetint concede the superiority of the present-value method in the special sets of circumstances described above, (where they allow that the circumstances exist at all), and there appears to be little disagreement as to the relative ease with which it can be utilized. There is one advantage claimed by Merrett and Sykes for the yield method but treated as an aside by Porterfield and Bierman and Smidt.[1]

Under a capital rationing policy where projects are not mutually exclusive, but are competitive, the present value measurement of worth does not reflect the influence of risk. This matter is illustrated in Table 3 which shows two competitive investment proposals and the incremental cash flows.

TABLE 3

COMPETITIVE INVESTMENT ALTERNATIVES, UNEQUAL COSTS

Project	Annual Cash Flow	Capital Cost	Life	NPV at 8%	Yield
A	$100,000	$502,000	10 years	$169,000	15%
B	144,000	780,000	10 years	185,000	13%
B-A	44,000	278,000	10 years	16,000	9.6%

Source: Merrett and Sykes, *op. cit.*, p. 154.

[1]Merrett and Sykes, *op. cit.*, pp. 154-155; Porterfield, *op. cit.*, p. 37; Bierman and Smidt, *op. cit.*, p.47.

The point the authors make is that the 9.6% return on the incremental cash outlay of $278,000 for the largest investment is significantly less than the 15% for the $502,000 investment. Therefore, if the firm's cost of capital is close to this incremental return, and the subjective risk factor manifests itself in some explicit "padding" of the cost of capital (i.e., an arbitrary cutoff rate of 10 percent for investments of this particular classification), then the project with the smaller present value but larger yield would be preferred.

This writer prefers Bierman's and Smidt's approach, wherein the relative desirability of the investment (as compared to not taking it) may be dramatized by the yield.[1] Assuming the present value is positive in all cases, if the yield is 35 percent for one investment, and other competing (as opposed to mutually exclusive) projects range from 12 to 30 percent, then the high-yield project appears very "safe," as well as technically profitable, and will be included to the possible exclusion of others with less "leeway." This consequence is most often manifested under a capital rationing policy.

However, the previously discussed profitability index (using the net present-value method) would provide the same relative visibility. Merrett and Sykes might interject their "tradition refrain" that businessmen are more accustomed to dealing

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[1]Bierman and Smidt, *op. cit.*, p. 47.

with yield than any index. The writer contends that the enlightened businessman will use what will serve him best.

Multiple Yields and Their Interpretation

Investments which give a multiple yield or none at all are known as nonconventional investments. An example of the latter situation would be where an investment has cash proceeds of \$100 and \$150 in Periods 1 and 3, respectively, with a cash outlay of \$200 in Period 2. This investment has no yield but does have a present value for all rates of interest.[1]

The reason for multiple yields is related to the formula used for both the yield and present-value methods. When solving for r as the unknown in the yield method, the analyst is solving for an unknown in a complicated polynomial equation:

$$0 = -C + \frac{A_1}{X} + \frac{A_2}{X^2} \cdots \frac{A_n}{X^n} \quad \text{where } x=(1+r)$$

There are as many roots (solutions for x) as there are years during which earnings are generated. Descarte's rule of signs for this type of equation limits the number of real solutions to the number of reversals of sign in the terms of the equation. In the convention investment with only one sign reversal, the yield method produces a single usable result. But if negative

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[1]Bierman and Smidt, *op. cit.*, p. 45.

earnings appear in the future cash flow stream, the method produced ambiguous results.[1]

When the formula is used in the present-value concept, the equation is simply a series of added fractions, since r is assumed. This method thus provides a simple algebraic additive process and provides a usable comparative evaluation of the projects regardless of negative or positive nature of the incremental cash flows.

An illustration is provided at this point to explain how a multiple yield should be interpreted and to show the relationship between the yield and present value under such circumstances.[1] The following Table 4 shows three series of cash flows: X, Y, and Z. X is a conventional one-year loan at 10 percent interest from the viewpoint of the banker (who gives and then receives). Y is the loan from the viewpoint of the borrower (who gets and then repays, with interest). Z is a multiple yield, incremental cash flow resulting from an algebraic comparison of period cash flows from a pair of mutually exclusive investment alternatives A and B (not shown).

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[1] Bierman and Smidt, *op. cit.*, pp. 41-44

TABLE 4

SERIES OF CASH FLOWS OF INVESTMENTS WITH MULTIPLE YIELDS

Investments	Cash Flows		
	Period 0	Period 1	Period 2
X	-100	+110	
Y	+100	-110	
Z	-72,727	+170,909	-100,000

Source: Bierman and Smidt, *op. cit.*, p. 42.

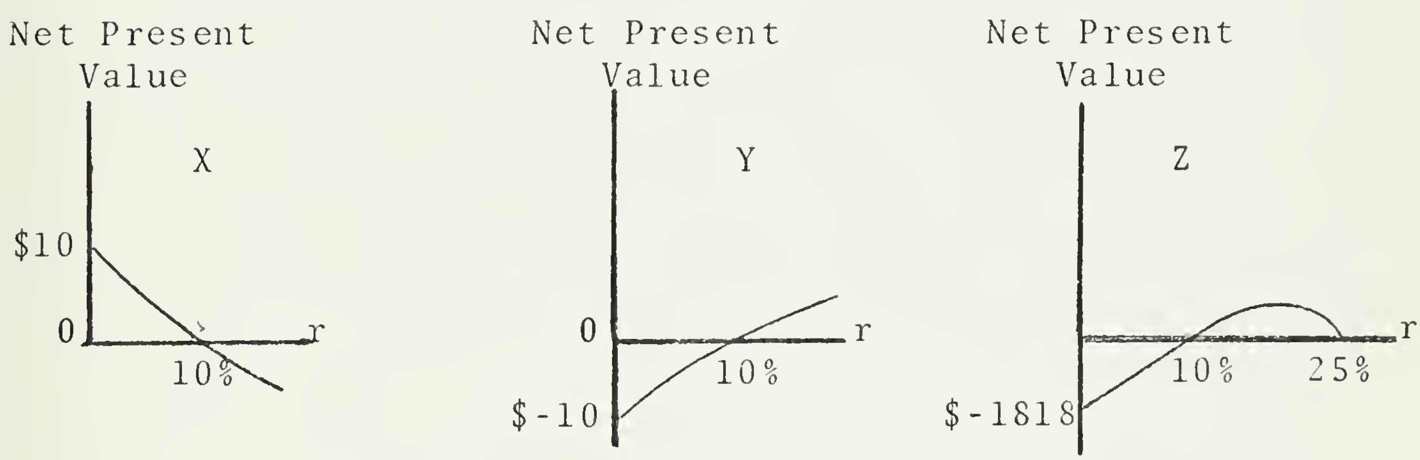

Figure 1 - Net Present Value of an Investment as a Function of the Interest Rate r .

Source: Bierman and Smidt, *op. cit.*, p.42

The yield of a cash flow was previously defined as the rate of interest that makes the net present value (NPV) zero. Thus, the yield is the point where the NPV line crosses the horizontal axis. The intercept is the rate of interest.

In Figure 1, X , the NPV line drops as the rate of interest increases. This typifies conventional investments in which cash outlays are followed by proceeds. The yield (or intercept) represents the highest rate of discount at which the NPV would be positive and the investment desirable.

Figure 1, Y , is inverted relative to X and indicates that from the borrower's point of view, the loan is worthwhile only if the rate of interest at which he finds it appropriate to discount future funds (which represents the real value of these funds to him) is greater than the rate of interest he pays on the loan. Thus, if he had to pay 10 percent interest on his loan, he would expect to be able to discount his future fund flows at something greater than 10 percent.

Figure 1, Z , is a composite of 1X and 1Y, the first part being typical of a loan; the latter having the downward slope of the ordinary investment. What the intercepts of this graph indicate are that the particular series of cash flows would be worthwhile at discount rates between 10 and 25 percent. Outside this range it is not advisable. Depending on the nature

of the original pair of investments compared, (A and B), one is superior for yields between 10 and 25 percent, the other for yields less than 10 and greater than 25 percent.

Figure 2 compares the two investments, where it is noted that investment A has a higher present value at rates of interest between 10 and 25 percent.

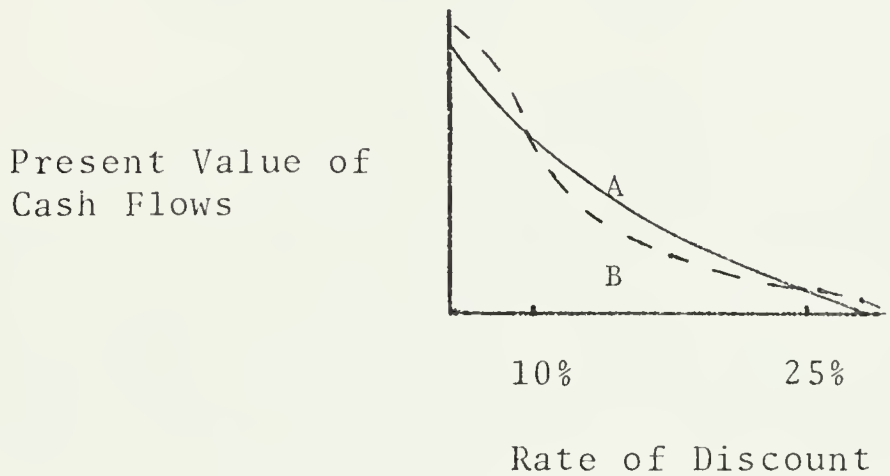

Figure 2 - Net Present Value of Alternative Investments A and B as Functions of Rate of Interest (Discount)

Source: Bierman and Smidt, *op. cit.*, p. 44.

However, in each case, a simple calculation of the net present value of the investment at the correct rate of discount would have provided a valid answer and obviated the problem of multiple yields.

While considerable attention has been devoted to this disparity of multiple yields, James Porterfield, for one, considers it a minor weakness. He states that projects with multiple or no real rates of return are relatively rare, resulting from unusual patterns of cash flows.[1]

[1]Pollock, *op. cit.*, p. 26.

The Reinvestment Assumption

The yield method implicitly assumes that proceeds are reinvested at the same return as the yield of the investments. The present-value method implicitly assumes that the proceeds are reinvested at the cost of capital. The fact that the latter assumption is more valid is one reason why the present-value method is preferred method.[1]

Contrary to statements sometimes made by proponents of other compound interest methods, the yield method involves no assumption as to the rate of return that can be earned on recovered capital.[2]

. . .the most significant part of a company's return is obviously the direct cash income from the investment, but this is not all. Another very important part is the incremental income gained by reinvesting that initial cash income. The very use by current methods of compound interest factors in discounting the direct income automatically provides for the reinvestment factor, but such incremental income is determined at the interest rate estimated for the proposal.[3]

(Some) have claimed that the system recommended... herein (the yield method) requires reinvestment of project earnings at the interest rate of return achieved on the project. It is the authors' belief that the analysis shown...makes clear that no reinvestment of profits at any interest rate is involved.[4]

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[1]Bierman and Smidt, *op. cit.*, p. 39.

[2]NAA, *op. cit.*, p. 62.

[3]Robert H. Baldwin, "*How to Assess Investment Proposals*," *Harvard Business Review* (May-June, 1959), p. 99.

[4]J. B. Weaver and R. J. Reilly, "*Interest Rate of Return for Capital Expenditures Evaluation*," *Chemical Engineering Progress* October 1956), p. 408.

...the mathematical manipulations involved in the calculation of (the rate of return) implicitly assume that all intermediate receipts, positive or negative, are treated as if they could be compounded at the rate being solved for.[1]

When we are evaluating the profitability of a project we are testing the earnings power of the money while it is invested in the project. What is done with the money after it is returned is completely irrelevant.[2]

...the rate-of-return (yield) criterion implicitly assumes that funds generated by a proposal can be reinvested at the same rate-of-return as the proposal itself offers, whereas the net prevent-value-per-dollar-of-outlay criterion (the present-value method) assumes that they can be reinvested at a rate equal to the company's cost of capital.[3]

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[1] Hirshleifer, *op. cit.*, p. 350.

[2] Raymond I. Reul, *"Calculating the Return on Proposed Projects, Equipment or Plant Facilities,"* Industrial Educational Institute, Boston, Mass. (Mimeographed), p. 6.

[3] Ezra Solomon, *The Management of Corporate Capital* (Glencoe, Illinois: The Free Press of Glencoe, Ill., 1959), p. 16.

The student of capital budgeting cannot be blamed if he becomes disoriented in this crossfire of diametrically opposed, authoritative viewpoints. However, Gerald Pollock claims that, ...*it can be demonstrated that a reinvestment assumption exists*, ...[1] The following exemplification borrows heavily upon his theory and example.

The argument starts with an example--one in which two investment projects are compared in Table 5.

Table 5

COMPARISON OF INVESTMENT ALTERNATIVES BEFORE REINVESTMENT OF EARNINGS

Project	Initial Investment	Net Cash Earnings First Year	Net Cash Earnings Second Year	Net Cash Earnings All Subsequent Yrs.
A	$100	$200	0	0
B	$100	0	$400	0

Source: Pollack, *op. cit.*, p. 13.

When the rate of return is computed for each project by the yield method, the results are identical--100 percent. The implication is that a $200 return received at the end of year one is just as profitable as $400 received at the end of year two. This further implies that the $200 received in year one can be reinvested at 100 percent rate of return (to obtain $400 in year two). Conversely, if the $400 of Project B earnings at the end of year two were to be converted to the $200, Project A earnings at the end of year one, the investor would have to borrow against the $400 at 100 percent interest. If he cannot do either

[1]Pollack, *op. cit.*, p. 13.

of these manipulations, the two projects cannot be considered equal. *The yield method implies that earnings are reinvested or borrowings occur at the same rate as that earned by an investment itself. Neither implication is reasonable.*[1] The cost of capital to a firm is a weighted and normally complex figure of several facets of its financial structure--rarely heavily influenced by a single project. Also, investments come in many forms and sizes. To assume that one investment will lead to another identical to the first is hardly a viable planning parameter.

The borrowing assumption is treated explicitly in the present-value method--being based on the cost of capital. Implicitly, earnings are reinvested at the same rate (the market cost of capital)--*a realistic assumption regarding the terms on which a company can actually obtain funds in the market.*[2]

Another example will illustrate the validity of this offsetting influence. With the cost of capital 10 percent, each of two projects require an initial outlay of $100, the earnings of which are as follows:

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[1] *Ibid.*

[2] *Ibid.*

Table 6

INVESTMENT ALTERNATIVES COMPARED AFTER REINVESTMENT OF EARNINGS

Project	Initial Investment	First Year	Second Year	All Subsequent Yrs.
D	$100	0	$363	0
E	100	$220	0	0

Source: Pollack, *Ibid.*, p. 14.

Applying the present-value method to Project D in Table 6, $363 discounts to $300 and provides a V/C ratio of 3.0, while Project E is rated at 2.0. If the first-year earnings of Project E were reinvested at some rate greater than 65 percent, it would appear that the projects were improperly ranked. However, the rational investor in Project D would borrow against the expected second-year proceeds up to $330 ($363 discounted one year) and also reinvest at 65 percent at the beginning of year two. At the end of year two, the expected gain would outpace that of Project D by a considerable margin. Therefore, if investors are able to borrow against future earnings, at the market rate, so as to make the most of investment opportunities that arise, project rankings do not change.

Under the yield method, however, borrowings are implicitly assumed to take place at the respective rates of return of the various projects under consideration. Thus, the relative profitability rankings of the various alternatives remain consistent

(given any rate of return on reinvestment) only if borrowings against the future earnings of any project occur at an interest rate equal to that project rate of return. It would be unlikely that a company would borrow at a rate above its cost of capital, and it cannot borrow for less. Therefore, borrowing on the assumed terms never occurs unless all the project rates of return happen to equal and coincide with the company's cost of capital.[1]

Summarizing, both methods do imply that earnings must be reinvested at certain rates. With the yield method, the reinvestment rate for earnings from a project is assumed to equal that of the project itself. The reinvestment assumption is, therefore, critical, since any modification of the reinvestment assumption may change the relative rankings of the projects involved. The present-value method normally uses the cost of capital as the reinvestment rate. Therefore, the reinvestment assumption is immaterial to the present-value method since modification of the assumption does not change the relative rankings.[2]

Despite the acknowledged complexity of determining the cost of capital,[3] a capital market exists, and the present-value method recognizes this fact and implies that firms will use it rationally.[4]

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[1] Pollack, *op. cit.*, p. 14.
[2] *Ibid.*, p. 16.
[3] Merrett and Sykes, *op. cit.*, p. 176.
[4] *Ibid.*

Summary

The present-value method is relatively easy and safe to use.

Based on the foregoing, where the two methods do not always rank the same projects in the same order of profitability, and where conflict exists, the present-value method provides worth measurement consistent with profit maximization.

The differences in relative rankings by the two methods stem from the implied reinvestment assumption that earnings must be reinvested at specified rates. While the validity of this assumption is not defended, its existence is. The import of the assumption upon present-value rankings is immaterial, but, in the yield method, it can be crucial.[1]

In an analysis of mutually exclusive projects, a comparison of respective yields will not necessarily give the optimum alternative. It is necessary to take two projects at a time, eliminate one, compare the survivor with another, and repeat until the best alternative is derived. The present-value method gives investment profitability in terms of magnitude and, if desired, in the form of a V/C index--all in one operation per alternative.[2]

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[1]Pollack, *op. cit.*, p. 16.

[2]Bierman and Smidt, *op. cit.*, p. 46.

In the rare but germane situation where negative and positive earnings are generated, the yield method gives multiple solutions; none of which may be an accurate measure of worth. The present-value methodprovides correct uncomplicated results in such situations.[1] In interpreting the single investment yield, it is necessary to identify the cash flows as that of an ordinary investment or of a loan from the point of view of the borrower.[2]

The yield method will indicate relative desirability among competitive projects from the standpoint of margin for uncertainty--an influence factor not provided by the present-value method.

Investment worth measurement techniques other than the present-value and yield methods were evaluated. Despite the wide use of some of these techniques in the business world, all were found deficient in certain aspects, primarily in that they did not recognize the time aspect of investment cash flows.

Other Techniques for Deriving Rate of Return

Before proceeding to investment cash flows and leaving the intussusceptive worth measurement methods, a discussion of some of return is considered appropriate. The assumptions underlying these simplified techniques span both those discussed under

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[1]Pollack, *op. cit.*, p. 17.

[2]Bierman and Smidt, *op. cit.*, p. 46.

the discounted cash-flow methods and the prime aspects of cash-flow analysis. In that light, they serve as a transition element.

New 1967 MAPI Method

This method, developed by the Machinery and Allied Products Institute, and authored by their Research Director, George Terborgh, is the third in a series of Institute presentations of a new system of investment analysis.[1] If used within the specification stated, it gives results close to those of the yield method.[2] Essentially, it provides a one-year comparison of investment in a project versus non-investment.

Comparison over a period of years (up to 10 years) is possible by averaging certain inputs. Within the ten-year time frame, with consistent operating advantage savings over the period, and after-tax returns not exceeding 15 percent, the MAPI averaging method gives a very close approximation to the yield method results.[3]

Forms which are provided with the text to assist the analyst, and thereby constitute the simplification aspect of the method, are based on two criteria--the comparison period (1 year or over 1 year) and the tax depreciation method. The MAPI form is designed for justification analysis, not for screening.[4]

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[1]Terborgh, *op. cit.*, p. iv.

[2]Murdick and Deming, *op. cit.*, p. 74.

[3]*Ibid.*, p. 75

[4]Terborgh, *op. cit.*, p. 42.

Profitability Index[1]

Raymond Reul, in his scheme, has simplified the "investor's Method" or "cash-flow method" through the use of forms and an easily followed computational sequence.

Five inputs are required to use the method:

(1) Total investment expenditures, plus a time schedule for them.

(2) Annual cost savings (returns) expected from the project.

(3) Useful life of equipment.

(4) Probable depreciation allowance.

(5) Expected income tax rates on profits.

Sample calculations aid the analyst in (1) compounding forward to present value all cash disbursements before "time zero" (when the investment is expected to start saving money), as indicated for four trial interest rates; and (2) discounting backward to present values disbursements and net profits (savings, or receipts less expenditures after income taxes but before depreciation) after "time zero," again using the factors indicated for four trial interest rates. The actual interest rate of investment is indicated by a 1:1 ratio found by using a special graph to plot the trial interest rates, then interpolating.

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[1] Raymond I. Reul, *"Newest Way to Figure Payoff," Factory,* October, 1955.

The author claims it provides the best-known "broad-gauge measure" for comparing economic worth of investments but emphasizes that care should be used in applying it--namely, that all factors should be considered and that..."*it's not a substitute for business judgment.*" The method can be used for buy or lease, act or not act, and in mixing problem decisions.

"Replaciation"

This term was adopted by B. A. Margo to characterize his simplified "guide to better decisions on machine replacement."[1] The author attempts to account for inflation and technological improvements in equipment by a "replaciation" vice depreciation method.

With the graphs and tables that typify "shortcut" methods for determining investment worth, the profitability of new equipment proposals can be determined if: (1) the life expectancy of the asset can be predicted; (2) if the general level of prices the time of replacement can be assumed.

The method can be illustrated with an example:

Assume a single machinery investment is contemplated at a cost of $10,000. The estimated life is 10 years. From experience, salvage is estimated at 10 percent of cost--or $1,000 in 1960 dollars. Normally, figuring only depreciation,

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[1]B. A. Margo, "*Latest Word in Profitability,*" *Factory,* January, 1958.

the target figure for net earnings would be $9,000 in 1960 dollars. However, price experience indicates inflation and machine improvements will increase the cost roughly 30 percent in 10 years. So, the replacement machine will cost an estimated $9,000 x 1.3, or $12,000. Thus, the annual replaciation rate is $12,000 ÷ 10, or $1,200 in 1960 dollars.

If predicted annual operating savings are $3,700 (1960 dollars), the replaciation ratio will equal $10,000 ÷ ($3,700-1,200)=4:1. With the graph provided, this ratio and the 10-year intercept (x axis) give a 21 percent return on investment on the y axis. Comparative profitability can be shown for other investment proposals estimated in the same way.

Future Value at the Terminal Date[1]

This approach can be categorized as a net present-value mirror image rather than a new concept, and thus while it does not fit the "shortcut" classification, it has been purposely left out of the discounted cash-flow controversy. Hunt Williams and Donaldson claim that it "is coming into favor."[1]

They use a schedule of cash flows of the corporate bond as the basis for their example. Assuming a return of 2-1/2 percent "opportunity rate" for the life of the bonds, they calculate

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[1]Pearson Hunt, Charles M. Williams, Gordon Donaldson, *Basic Business Finance: Texts and Cases* (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 432-433

the total value at the terminal date, having reinvested at the 2-1/2 percent rate (compounded) the periodic cash receipts. This total future value is a single number, like the present value, but at the terminal date of the investment, vice time zero. Subtracting the net price of the bond from the calculated future value, they derive a growth in value. A rate of interest is then calculated that would produce this growth in value. This rate is compared with the "opportunity rate" or 2-1/2 percent to obtain a "go/no-go" sequel; i.e., any rate above 2-1/2 percent is "go."

CHAPTER III

EVALUATION OF THE CAPITAL INVESTMENT

In the preceding Chapter, several methods for calculating investment worth were analyzed. It was determined that any acceptable system of investment evaluation should consider both the amount and the timing of net cash outlays for and net differential cash inflows from the investment. Based on this hypothesis, only the discounted cash flow techniques were found technically sound. Time, in effect, is the dimension through which the monetary variables involved in investments--*the capital outlays and subsequent net receipts*--must be related.[1] It follows that cash flow forecasts in terms of actual disbursements and receipts are the very essence of discounted cash flow measurement methods, and that the methods are only as useful as the relevance, accuracy and comprehensiveness of the data supplied. The purpose of this chapter is to define the nature of cash flows and explain the nature of the problems inherent in applying a cash-flow analysis to investment proposals.

Cash flows are not necessarily income and expense in the accounting sense of these terms.[2] Outlay can range from maintenance to acquisition of fixed assets--but still figure

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[1] Pollack, *op. cit.*, p. 2.

[2] Bierman and Smidt, *op. cit.*, p. 89.

in the project analysis. Cash proceeds can be tax savings as well as net operating revenue. These flows are related to the investment to derive return on investment in one form or another.[1]

According to Bierman and Smidt, the cash flow procedure avoid several problems associated with accrual accounting such as:[2]

1. In what time period should revenue be recognized?

2. What expenses should be treated as investments, capitalized and depreciated over n time periods?

3. What method of depreciation should be used in measuring income for internal and external reporting purposes (as opposed to income measurement for tax purposes)?

4. What sort of inventory measurement system should be used?

5. What costs are "inventoriable"?

To the uninitiated, the cash flow concepts and calculative processes might appear to fly in the face of accepted accounting practice. In reality, the former is part of capital budgeting; while the principles and procedures related to the preparation of financial statements have the underlying purpose of assuring a fair statement of all revenues and expenses for the respective *accounting period* and/or a fair statement of all assets and equities as of the last day of that period.[3] The decision on

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[1] *Ibid.*

[2] *Ibid.*

[3] C. Rollin Niswonger and Philip E. Fess, *Accounting Principles* (Cincinnati, Ohio: South-Western Publishing Co., (1965), p. 197.

whether to undertake a proposed capital project entails the determination of the amount and timing of *net cash outlays* and *inflows*, rather than *net income* as computed in the conventional accounting sense.[1] While accounting methods for income determination and the planning and control of operations primarily have a current time-period orientation, capital investment planning has a project orientation that normally entails commitments over a period of time greater than one year.[2] Thus, there is a danger in using ordinary accounting data for special purposes such as investment decisions. Special techniques, such as the discounted-cash-flow method are required for long-range planning because the time-value of money becomes extremely important when investments extend beyond one or two years. From the standpoint of the analyst, the overriding goal of investment is maximum long-run net cash inflows.[3]

While the cash flow procedure avoids certain complications, it introduces procedural and conceptual aspects that in themselves are involved and often perplexing. However, before introducing these aspects, it is considered appropriate to define some terminology used in the cash-flow process.

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[1] Edward J. Mock, Robert E. Schultz, Raymond G. Schultz, and Donald Hart Shuckett, *Basic Financial Management: Text, Problems, and Cases* (Scranton, Pa.: International Textbook Co., 1968). pp.184-186

[2] Charles T. Horngren, *Accounting for Management Control: An Introduction* (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1970) p. 168.

[3] *Ibid.*, p. 371

1. *Cash Flows*, or the more common term, *net cash flows*, associated with a specific investment can be defined as *incremental* cash receipts and expenditures solely attributable to the undertaking of the project. Cash outlays, or negative cash flows, normally occur at the commencement of a project, but also occur when capitalized items are replaced during the project life. Terminal remedial measures such as reforesting and operating losses also comprise cash outlays. Cash receipts or positive net cash flows comprise the incremental cash inflows such as profit, rent and depreciation.[1] These two types of net cash flows are further defined below.

2. *Capital Outlays*, also known as *negative net cash flows*, must be carefully analyzed so that only incremental outlays are attributed to the investment in question.[2] For instance, a factory building to house two net products and increased production of an existing product should not be charged to one or two of the projects but, rather, to all three in proper proportion. Conversely, if the entire building is to house a single undertaking, it should be charged in toto to the project.

When assets already owned are converted for use in the new project, their "worth" must be assigned as an outlay. This valuation seldom equates to the book value of these assets but, rather, is quantified as the actual sacrifice the firm makes through assigning the assets to the project,[3] at times a highly subjective consideration.

Another complexity in the capital outlay estimation involves valuing shares issued by the firm for the purpose of financing the project. This is a complex matter, beyond the scope of this chapter. Suffice to say that there is a need to value this means of payment and incorporate it into the investment outlay analysis.

3. *Cash Receipts*, or net cash flows comprise the net cash inflows produced by the investment. Essentially, they comprise profits less taxes when actually paid, plus the depreciation provisions less replacement capital expenditures when actually made, plus net changes in working capital, plus recovery of any net residual value from assets at the end of the investment life.[4]

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[1] Merrett and Sykes, *op. cit.*, p. 43

[2] *Ibid.*

[3] *Ibid.*

[4] *Ibid.*, p. 44

In accord with most authorities on this matter, profits are considered net income, or cash after taxes, and depreciation is included as part of the net cash flow. The bases for these arguments are discussed in detail later in this chapter.

4. *Schedule of Cash Flows* is often referred to as an initial step in working up a capital budget. It is the projection of cash inflows and outflows, as defined above, together with the time dimension of cash flow. According to Hunt, Williams and Donaldson, six elements comprise such a schedule.[1]

a. The estimated amount of the net capital investment required to implement the project.

b. The amount of value expected to remain at termination of the project life.

c. The estimated life of the project.

d. The selective return (i.e., comparison to the benefits of investing in an alternative project).

e. The timing of cash flows.

f. The change anticipated in operating funds flows of the firm attributable to the project.

The matter of *investment* or capital outlay was previously defined. One distinction is pertient--that of initial and subsequent investments. While the first may be easily determined, if the project in its lifetime is to recover all capital outlay, it is essential that all subsequent capital outlays be recognized and included in the analysis. The desired figures are the net of all related movements, such as start-up costs and disposal proceeds.[2]

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[1]Hunt, Williams and Donaldson, *op. cit.*, p. 413.

[2]*Ibid.*, p. 415

Terminal Values or residual values are what is recoverable from the project at the termination of its life, such as through resale or employment elsewhere. They should be the net of removal costs, tax allowances or charges, and miscellaneous expenses involved in the sale and/or disposition of the project facilities and equipment.[1] They must be considered by the analyst since the firm looks for recovery of its investment from liquidation of the assets assigned the project as well as proceeds from operations.[2] The terminal value estimate, of course, is dependent upon the time at which the project is expected to be discontinued.

Economic life is normally the time regarded as the productive life of the project--the period over which the investment remains economically superior to alternative investments that could be acquired for the same purpose. It is the time between acquisition and obsolescence.[3] The life estimate can be as difficult as any but, nonetheless, must be made, since the project justification rests upon it. In this aspect of cash-flow analysis, experience would seem as valuable as any analytical technique.

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[1] Merrett and Sykes, *op. cit.*, p. 50

[2] Hunt, Williams and Donaldson, *op. cit.*, pp. 415-416.

[3] *Ibid.*

Returns and Timing are fully explored further on in this Chapter. A special adaptation of the previously discussed opportunity cost is sometimes employed in comparative analysis for certain types of investments. An opportunity rate is used, such as the after-tax yield on an assortment of bonds.[1] The rate of return of the contemplated investment is judged against the opportunity rate to test potential profitability.

Change in operating funds flow is the difference between what the total cash received from operations would be if the change is not made and what cash the firm is expected to receive if this project is adopted.[2] When analyzing investment cash flows, this effect on working capital must be considered. As an investment ties up funds, more cash is needed for day-by-day operations. Hence, a working capital increase has the effect of increasing the investment outflow today. Ignoring this factor will lead to acceptance of investments which should be rejected. In the interests of financial analysis, it is essential to plot all cash flows, recognizing quantitative irregularities and their timing, "even though all must recognize that estimates of this kind are sure to be inaccurate."[3]

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[1] *Ibid.,* p.417.

[2] *Ibid.,* p.415.

[3] *Ibid.,* p.416.

The element of uncertainty also plays a large hand in certain types of investment. For instance, a project to introduce a new product entails some uncertainty of market acceptance. Even if acceptance does occur, the time element from initial production to significant demand can vary considerably, as can the degree of linearity of such response. A hypothetical plot of revenue from the time of product introduction, through accelerating demand, and finally through a negative gradient as competitive products emerge is presented as curve (a) in Figure 3. Curve (b) represents sales of a product made "prematurely" obsolete by competitive entries with innovations and/or superior technology.

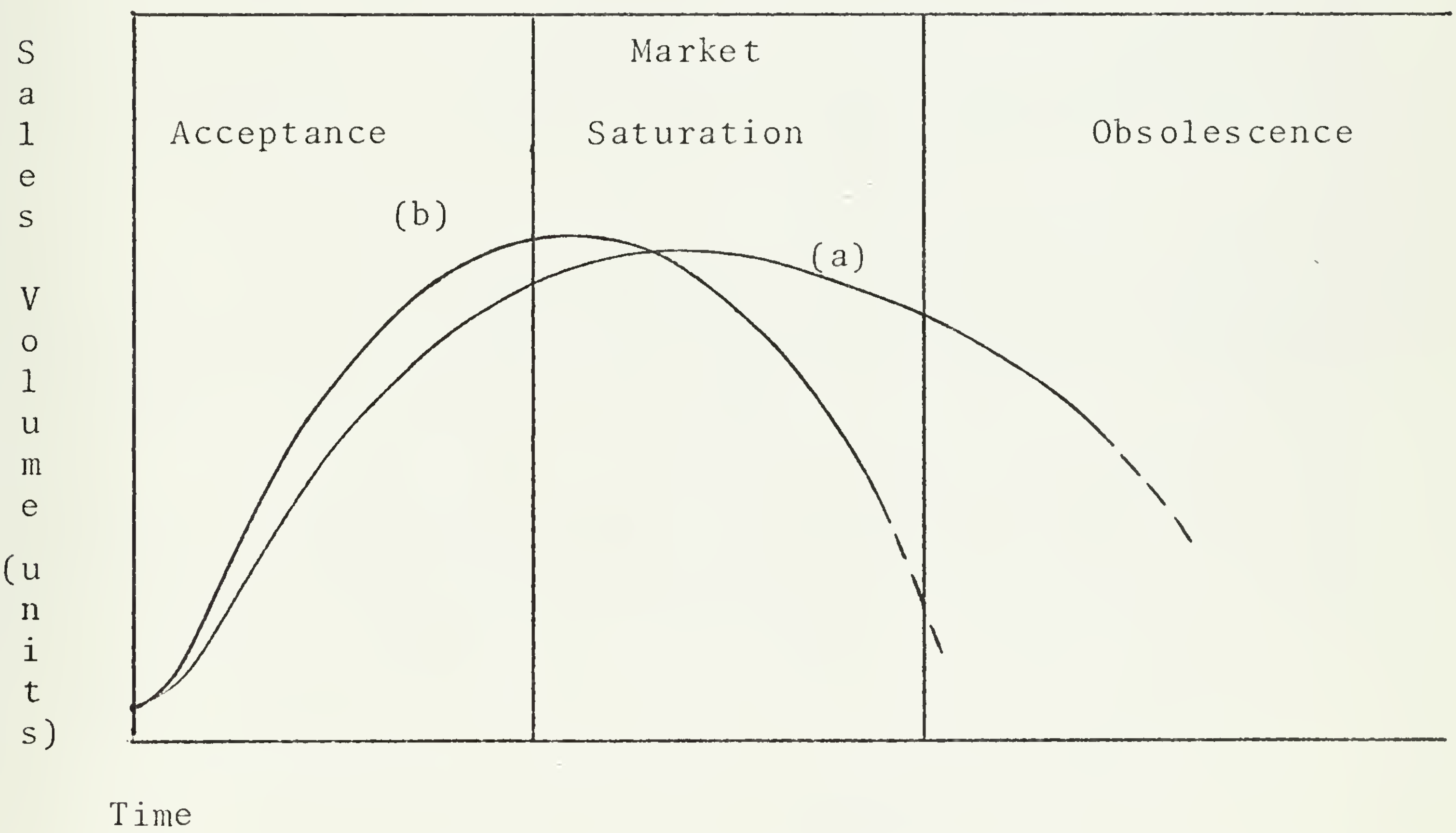

Figure 3 - STAGES OF MARKET DEVELOPMENT FOR (a) SUCCESSFUL PRODUCT, AND (b) PRODUCT AFFECTED BY EARLY OBSOLESCENCE

Source: Thomas A. Staudt and Donald A. Taylor, *A Managerial Introduction to Marketing* (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1970), p. 168.

Net cash flows are also influenced by such factors as maintenance and labor costs, which can be expected to rise with time. Terborgh emphasizes the need to investigate specific changes in prospect for all categories of cost, such as fringe benefits, tooling, maintenance, materials and supplies, inspection, assembly, scrap, utilities, down-time and subcontracts.[1] Complicating the calculations are indirect or secondary costs and benefits. He also differentiates between effects on revenue and on costs, accentuating the need to appraise the import of the project on revenue in terms of output and product quality.

The procedural framework for estimating cash flows from operations of an investment will vary somewhat from business to business; however, regardless of sequence and refinements, the following basic operations apply to most investment cash-flow compilations.[2]

1. A sales figure is compiled representing the dollar-value of sales expected to be recognized during the period concerned. It is adjusted for timing of receipts and decline (or increase) in sales of other affected products.

2. Production costs (including assigned overhead) are estimated for the same period for the actual rate of production (not sales) expected during the period. This includes all costs peculiar to the investment.

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[1]Terborgh, *op. cit.*, pp. 139-141

[2]Bierman and Smidt, *op. cit.*, pp. 176-196.

3. Adjustments are made for changes in working capital, with the same procedures used in preparing cash budgets. Any increase should be the amount estimated for the support of the investment operations.

4. Further adjustments are made for cash flows in other parts of the business resulting from the investment.

5. Tax adjustments are made (before depreciation), often from a form arranged like an income statement. From this figure is subtracted the tax savings from depreciation (again, normally calculated using a form).

6. After-tax cash flows for each year are then obtained by subtracting the discounted algebraic sum of costs and adjustments from the discounted sales forecasts.

There are some investments, such as a new cafeteria, or research where cash flows in terms of benefits are not easily evaluated. New techniques of measurement have been introduced that have helped quantify certain nebulous aspects; but it must be recognized that some investments are not subject to the type of analysis presented in this paper.[1]

No further treatment will be given the technical aspects of cash-flow analysis other than to summarize the views of most authors in the field of capital budgeting. Their near-unanimous approach to the procedural aspects of estimating cash-flows is the use of forms which, by format and sequence, guide the analyst through the necessary procedural steps to insure inclusion of all pertinent data and recognition (conscious or otherwise) of underlying concepts. A capital budgeting manual is a common instrument for policy guidance, investment classification, and administration, and often provides the theoretical

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[1] Hunt, Williams and Donaldson, *op. cit.*, p. 416.

basis for the scenario on the forms.[1]

There are three conceptual aspects of cash-flow analysis of which the analyst should be cognizant. These are: (1) investment timing; (2) the theory of relative and absolute returns; and (3) the implication of taxation. While timing may be considered an integral part of any investment consideration, it has been addressed separately here for reasons of clarity and emphasis. It reappears in typical interaction fashion in the subsequent discussion on returns and taxes.

Timing of Investments

Investment decisions typically involve two questions: *What* investment and *when* to implement it. Within the *when* or timing aspect there are two policy questions to answer. (1) Is the timing correct for the investment threshold the company is using? (2) Is the threshold (or cutoff) itself correct? Assuming the latter is correct, a replacement project should be implemented when its relative return (the return relative to the alternative of going on without it for the best comparative period) equals the cutoff rate. At this point, there is no further period of deferment for which it is economically feasible to go on without it.[2]

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[1]See Terborgh, *op. cit.*, Chapters 13, 14 and 15: Bierman and Smidt, *op. cit.*, Chapter 13; Murdick and Deming, *op. cit.*, Appendix; Merrett and Sykes, *op. cit.*, Chapter 14.

[2]Terborgh, *op. cit.*, p. 309

Whatever the need for investment, whether it be functional or material obsolescence, mistiming can be expensive "in terms of avoidable costs incurred or revenue foregone."[1] Ironically, when a need for investment is obvious, it may already mean that the investment decision is late. This problem is particularly common where obsolesence has suddenly, unexpectedly confronted management. Yet, there is considerable risk in replacing a current operation with a second-generation equipment when development of a third-generation is underway. The dichotomy of prolonging acknowledged obsolescence because of uncertainty of future obsolesence of a replacement is not a new quandry to many industries.

Alchian and Terborgh advocate assuming a rate of technological change based on historical trend.[2] Bierman and Smidt take exception to this method and espouse the use of specific information about the future and the probability of technological improvement.[3] The basis for rejecting rate of change as a parameter is that specific information is more pertinent; and, where there is a paucity of specific data, they advocate ignoring the timing problem, assuming a constant rate

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[1] *Ibid.*, p.310

[2] See A. Alchian, *Economic Replacement Policy*, The Rand Corporation, (Santa Monica, California, 1952); and G. Terborgh, *Business Investmen Policy*, (MAPI, Washington, 1958.)

[3] Bierman and Smidt, *op. cit.*, p. 79.

or using a range of estimates.

Whatever the philosophy, differential timed investment decisions can produce different cash flows because of changes in costs, revenues, or both--a fact the analyst should appreciate. It is assumed that the analyst's job is to find which investment will maximize the present value of the cash flows.

In this regard, there are at least three basic choices of action:

1. Wait to invest until the current equipment or facility reaches the end of its economic life. (In other words, no definite investment plan in the near future).

2. Plan to invest at some definite but future time before the end of the economic life of the current equipment.

3. Invest now, with the possibility of replacing the new equipment at the end of its economic life.

The example below is used to demonstrate the effect of timing decisions on the investment cash flow.

The following data is assumed:

--The present and replacement equipment (brine pump) are identical;
--the physical life is four years with no salvage value at time of replacement;
--cash flow estimates are after-tax;
--the present pump is one year old;
--cost of capital to the firm is 10 percent.

Outlays and proceeds for the current and replacement pumps are shown in Table 7.

Table 7

COMPARISON OF THREE ALTERNATIVE COURSES OF INVESTMENT

Pump	Initial Outlay	Estimated annual net cash proceeds
Continue with pump	-	$ 20
Replace now	$236	100
Replace one year hence	200	100

Source: Bierman and Smidt, *op. cit.*, pp. 80-84, *passim.*

Three hypothetical courses of action will be considered:

Plan A -- Replace three years from year zero (now).
Plan B -- Replace now, use equipment for entire life, then replace again.
Plan C -- Defer replacement for one year.

Using the above data and assumptions, Table 8 shows the cash-flow schedule for each plan.

The next step is to compute the present value for each plan [see Table 9] with the largest value representing the best alternative. The following notes pertain to Table 9.

1. The $63.09 per period is equivalent to $200 every four periods.

2. The factor 10 appearing in each computation is the present value of a dollar per period for an infinite number of periods assuming a rate of discount of 10 percent.

3. The present values are being computed as of the end of period 0 or the beginning of period 1. The investment and cash flows of each period are assumed to take place at the end of the indicated period.

Table 8

COMPARISON OF YEARLY CASH FLOW FOR ALTERNATIVE COURSES OF INVESTMENT

Investment	Cash flows at end of year								
Plan	0	1	2	3	4	5	6	7	8
A ...	$ 20	$ 20	$ 20	$ 20 [$200]	$100	$100	$100	$100 [$200]	$100
B [236]	100	100	100	100	100 [200]	100	100	100	100 [200]
C ...	20 [200]	100	100	100	100	100 [200]	100	100	100

Note: Outlays to purchase pumps are bracketed.

Source: Bierman and Smidt, *op. cit.*, p. 81.

4. Instead of converting the $200 outlay every four years to an annual equivalent of $63.09, it is also possible to convert the annual interest of 10 percent to an equivalent interest of 46.4 percent over a 4-year period, and compute the present value of a perpetuity of $200 per period (each period, 4 years) using the 46.4 percent.

Plan A is the least desirable, the decision to wait having reduced the size of the perpetuity of annual net cash proceeds without a compensating reduction in outlay for perpetuity.

Plan C has avoided the relatively high immediate outlay of Plan B and undertaken the investment the following year, producing the calculated operating advantage over Plans A and B.

The example assumes different cost outlays for investments at the end of periods zero and one, yet no change in subsequent investments--a perhaps unlikely happenstance. However, additional plans could be evaluated to suit any change in this assumption.[1]

The brine pump, by its very function, has a fairly predictable life. Being a relatively simple mechanism, maintenance and obsolescence are not primary factors; nor is salvage or terminal value. There are machines, however, that will last for decades (provided the vulnerable parts are replaced as needed), and that will retain a salvage value more or less inversely proportional to its time in service. Here, again, the replacement

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[1] Bierman and Smidt, op. cit., p. 84.

Table 9

COMPUTATION OF PRESENT VALUE OF CASH FLOWS FOR THREE ALTERNATIVE COURSES OF INVESTMENT

	Present value of cash flows	Explanations
Plan A		
$20\ (2.4869) =$	$ 50	$20 a period for three periods
$100 \times 10 \times (1.10)^{-3} =$	751	A perpetuity of $100 discounted for three periods
$\frac{200}{3.170} \times 10 \times (1.10)^{-3} =$	(474)	The annual equivalent outlay for perpetuity discounted for three periods
	$ 327	Net present value for Plan A
Plan B		
$236.00 =$	$ (236)	An immediate outlay
$100 \times 10 =$	1000	Cash inflows of $100 for perpetuity
$63.09 \times 10 \times (1.10)^{-4} =$	(431)	The annual equivalent outlay for perpetuity discounted for four periods
	$ 333	Net present value for Plan B
Plan C		
$20\ (1.10)^{-1} =$	$ 18	$20 discounted for one period
$100 \times 10 \times (1.10)^{-1} =$	909	A perpetuity of $100 discounted for one period
$63.09 \times 10 \times (1.10)^{-1} =$	(574)	The annual equivalent outlay for perpetuity discounted for one period
	$ 353	Net present value for Plan C

Source: Bierman and Smidt, *op. cit.*, p. 82.

timing affects cash flows, and hence the optimization of the investment.

As an example, assume that a 4,000-pound capacity forklift truck costs $4,000 new and that this type of material handling equipment as the upkeep and salvage experience record shown in Table 10.

Table 10

ESTIMATED REPAIR AND MAINTENANCE COSTS FOR EQUIPMENT FOR EACH IN SERVICE

Years in Service	Repair Costs	Maintenance Costs	Salvage Value at end of year
1	600	100	3,000
2	1,200	150	2,000
3	1,300	200	1,000

Source: Adaptation of examples in Bierman and Smidt.,*op cit.*, pp. 85-86.

Table 11 represents this data as a cash-flow schedule (assuming that repairs are not done the year of sale but that maintenance has been).

Table 11

EQUIVALENT COST PER YEAR OF ALTERNATIVE EQUIPMENT REPLACEMENT SCHEMES

Replacement	Cash Flows for Year*						
Period (year)	0	1	2	3	4	5	6
1	(4000)	(4000)	(1100)	(1100)	(1100)	(1100)	
		(100)					(100)
		3000					3000
2	(4000)		(4000)		(2150)		
		(100)	(150)	(700)		(700)	(150)
		(600)	2000				2000
3	(4000)			(4000)	(700)	(1350)	
		(100)	(150)	(200)			(200)
		(600)	(1200)	1000			1000

*Lowest common multiple of years.

Source: Adaptation of examples in Bierman and Smidt, *op. cit.*, pp., 85-86.

Discounting these flows at an assumed cost of capital of 10 percent, the present values of the three alternatives are:

Replacement Period	Present Value of Costs for Six Years
1 Year	$6535
2 Years	7791
3 Years	9051

Annual replacement (as practiced by some rental car agencies) is the most profitable way to maintain a fleet of this type of materials handling equipment. Hanging on to "brand new, hardly a scratch" equipment would, under just these circumstances, reduce profitability.

There are cases where beneficial projects have been available but, through poor research practice, or administrative delay, have been left dormant. As previously pointed out, this failure to invest at the optimum time can dilute earnings. However, a company ignorant of the subtle but trenchant implications of mistiming may find its investment planning in serious trouble. "In many decisions, the actual timing of the investment itself may have more to do with its ultimate success or failure than any other aspect of its handling."[1]

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[1] Chester R. Wasson, *The Economics of Managerial Decision* (New York: Appleton-Century-Crofts, 1965), p. 176.

The tax implications of investment timing can be significant in projects where a part of the outlays are expensed against current income for tax purposes. If the tax rate is expected to rise, investment deductions from future revenue will improve net income more than the same item expensed against current revenue.[1]

To capitalize, managing the investment timing problem on a continuous and full-scale basis, presupposes familiarity with the insidious effects of mistiming and the utilization of every scrap of pertinent information. Thorough analysis of musty records and bulging operational data files, particularly where technological uncertainty is a factor necessary to enhance the judgment and inevitable intuition of the investment decision. However, regardless of the data available, decisions must be solved by considering all alternatives and choosing those that maximize the present value of the cash flows.[2]

Absolute and Relative Returns on Investments

A project can have two kinds of return. "This distinction is basic to an understanding of investment analysis."[3]

[1] Dean, *Capital Budgeting,* p.20.

[2] Bierman and Smidt, *op. cit.,* p. 87.

[3] Terborgh, *op. cit.,* p. 55.

The *absolute* return or cash flow is that which comes from the project operating margin--the net cash flow of its revenues and operating costs. The *relative* return is the difference between the net cash flows of the project and that of an alternative. The former is not always available, but the relative return can be figured wherever there is an identifiable alternative to the project. Since a project has but one net flow of revnues and costs, there is not but one absolute return. On the other hand, there can be as many relative returns as there are valid alternatives.

This matter can best be seen through the use of a graphical presentation which portrays the distinction of absolute and relative returns and incorporates the aspect of timing as discussed in the prior section.

In Figure 4, absolute earnings are shown in the area ABC, relative earnings in ADEC. The absolute flows diminish with time, as revenue falls; whereas, the relative earnings remain constant. (It would be no task to show the more likely condition of steadily increasing costs, rather than stable costs, which would further reduce the absolute earnings area. The effect on relative earnings would depend on the level of costs without the project). It is to be noted

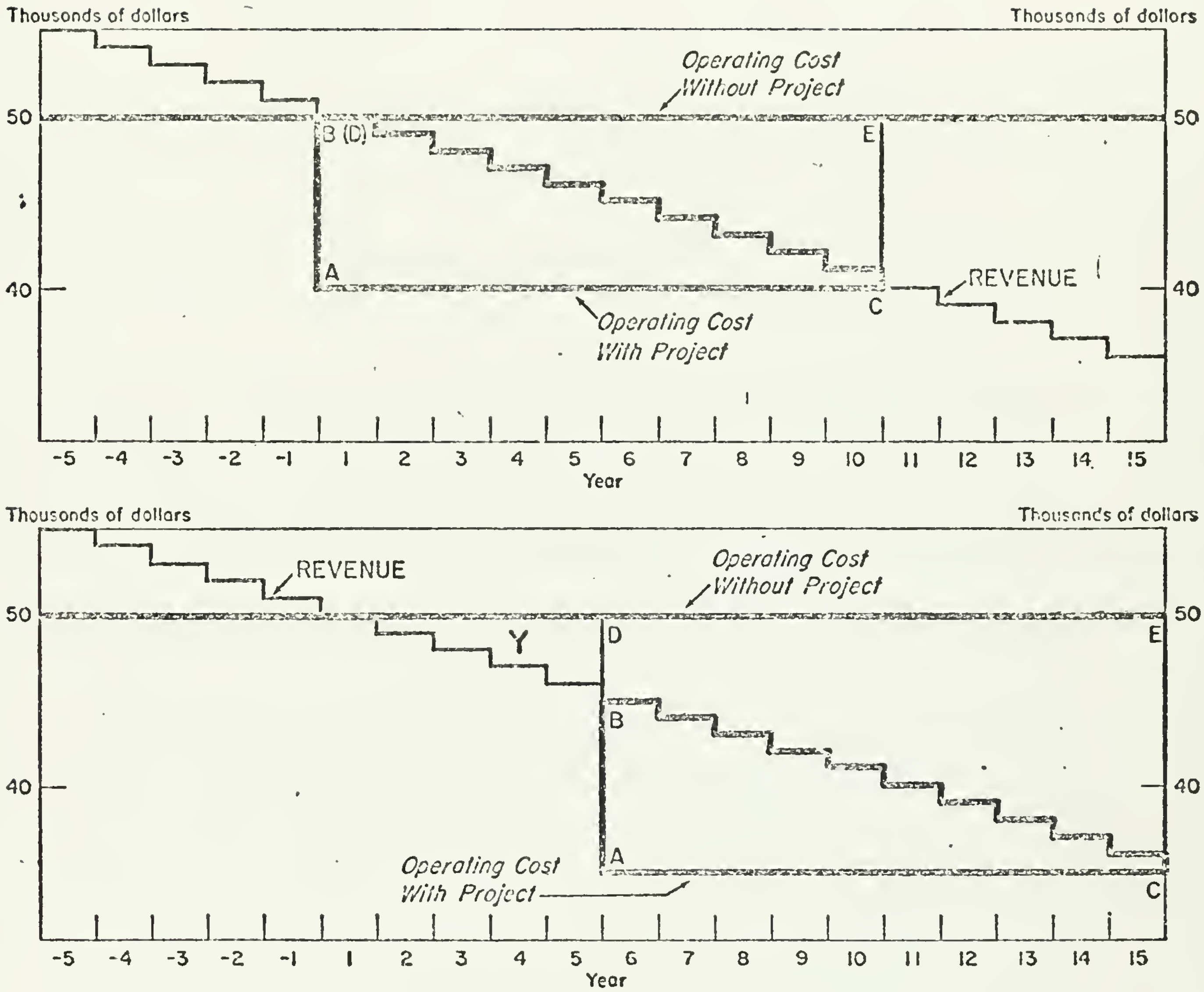

Figure 4

Absolute and Relative Earnings of a Hypothetical Project, for Analyses Made at the Beginning of Years 1 (A) and 6 (B). A Case of Obsolescence

Source: Terborgh, *op, cit.*, p. 62.

that the absolute earnings are the same for both alternatives, A and B, but the relative earnings for B are $15,000 vice $10,000 for A. This additional $5,000 in B stems from the fact that the alternative was operating at a $5,000 loss per year when the project was implemented. Area Y represents earnings foregone because the project was undertaken five year late.

These graphs also point up some conceptual dangers in the use of the relative return. It affords no criterion for the proper timing of the investment. In the example given, procrastination would lead to greater profits. This fallacy may blind management in certain analyses and be responsible for the retention of equipment well beyond its economic life--a costly misconception.[1] Also, as previously explained, an estimate of relative returns (or cash flows) entails a comparison of two alternatives. If the alternative is a poor one, the new project looks like a very bright prospect. "This means that almost any investment can be made to seem worthwhile if it is compared with a sufficiently bad alternative."[2] The solution is to consider all alternatives. In doing this, and picking one alternative as standard, the analyst obtains the selective picture he needs as a decision aid.

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[1]Terborgh, *op. cit.*, p. 69.

[2]Bierman and Smidt, *op. cit.*, p. 93.

With a series of absolute cash flows, it is possible to compare actual results with those estimated, period by period. There is no such similarly identifiable series of cash flows with which to compare relative cash-flow estimates.[1]

Why then are relative earnings (or returns or rates of return) used, if they pose such problems--particularly since it is difficult to make ultimate use of cash flows that are a byproduct of the difficulties of estimating the incremental effects of various actions of the firm? The answer normally lies in the reality of business life. When absolute returns are available, they tend to ignore "interaction effects."[2] For instance, an investment is made for the purpose of manufacturing a new product. The estimated absolute return is 10 percent on the $1,000,000 investment. However, the product helps to "round out" the company product line, improving its competitive market position. It also provides a technological base from which the company will be able to move into associated but new market areas. Some older less profitable items can be dropped, while better commonality is achieved in material orders and parts stock. The net effect of these interaction benefits is to improve the company-wide profit forecast by $50,000. So, while the investment profits remain the same,

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[1] *Ibid.*, p. 92.

[2] Terborgh, *op. cit.*, p. 65.

and the absolute return is 10 percent, the company benefits increased by $150,000, making the relative return 15 percent.[1] Ignoring collateral benefits may lead to rejection of sound investment opportunities. On the other hand, failure to consider such "upstream and downstream" effects may lead to unwise selections.

When a project is completely independent, such as a new business, there may be no interaction effects, and the absolute return may be a satisfactory measurement of worth. However, where there is a net interaction, the absolute return will under- or over-state the merit of the investment. On the other hand, the timing problem may require the computation of absolute as well as relative returns, for optional timing depends on their interrelation.[2] This was illustrated in Figure 4.

The present value of a series of relative cash flows will be the same as the present value of absolute cash flows of alternatives subtracted one from the other--if the absolute cash flows are adjusted for interaction effects. The net present-value method will lead to the same conclusion, whichever approach is used.[3]

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[1] *Ibid.*, p. 65.

[2] *Ibid.*, p. 69.

[3] Bierman and Smidt, *op. cit.*, p. 92.

One last aspect of relative/absolute returns is considered germane before proceeding to taxes and depreciation in cash-flow analysis.

When absolute returns are computed for a business investment, the period used is normally the investment or project life. In some instances, such as an underground water line, it may not be practical to use such a period, which may be anywhere from fifteen to fifty years. In such cases, a "study period" may be used, which is a substitute for the full life of the project. This study period then becomes the basis for comparison.

In the relative return analysis, the return is derived from the difference between two alternatives, and the analyses period is the period of comparison. In this case, the period used is the life of the alternative. The alternative may be either some other project or a decision not to invest. The rationale for this criterion is that since the relative return is based on the difference between an investment proposal and its alternative, the analysis cannot properly be made beyond the use life that is common to both projects. This qualification would not hold in the unlikely case that the alternative had a longer life than the project.[1] However, a

[1]Terborgh, *op. cit.*, p. 104.

common service life would still be necessary as a basis for computing relative returns.

Income Taxes and Cash Flows

With tax rates as high as they are in this country, it is not surprising that the tax aspect is a prime consideration in most investment decisions. As explained previously, taxes affect both the timing and the amount of cash flows.[1]

Depreciation is one primary means through which taxes affect cash flows. The method of depreciation determines the amount of the asset cost that is expensed each year, and, hence, the cash flow that is shielded from the "tax bite."

The book value of an asset is essentially irrelevant to the question of asset replacement. (It is a sunk cost and as such should have no bearing on the decision.) However, when an existing asset is disposed of, there may be a gain or loss on the remaining book value, which is subject to tax laws and, hence, cash flows are altered.[2]

These aspects are portrayed along with a complicating but common problem of unequal lives in the example that follows.

> *The problem stems from the fact that the existing equipment has an estimated remaining life of four years, while the expected life of the new machine is 10 years.*

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[1]Bogen, *op. cit.*, p. 17.41

[2]*Ibid.*

If the machine is replaced now, a comparison can be made as of the end of four years when the useful life of the old machine is terminated. But, at that time, the replacement machine will have a life expectancy of six years, and a considerable salvage value that must be taken into consideration.

For analysis, Tables 12 and 13 relate the data in tabular form. As illustrated in Table 12, machine A is completely written off at the end of four years (ten years of service at $1,000 per year depreciation). The disposal value is, therefore, a capital gain. Had the machine been disposed of at year nine for $1,000, there would have been no capital gain or loss, with the salvage value just equating to the $1,000 remaining book value.[1]

Table 12

COMPARISON OF
INVESTMENT ALTERNATIVES WITH DIFFERENT USEFUL LIVES

Machine A	
Cost new, estimated life 10 years	$10,000
Book value now (straight-line depreciation)	4,000
Remaining useful life	4 years
Disposal value now	2,500
Disposal Value 4 years hence	1,000
Annual Operating Costs, exclusive of depreciation	10,000
Minimum desired return	12%
Machine B	
Cost new, estimated life 10 years	$20,000
Disposal value 4 years hence	12,000
Disposal value 10 years hence	4,000
Annual operating costs, exclusive of depreciation	3,000

Source: Bogen, *op. cit.,* 17.43, 17.46.

[1] *Ibid.,* p. 17.46.

Table 13

COMPARISON OF PRESENT VALUE OF CASH FLOWS
FROM ALTERNATIVE INVESTMENTS WITH UNEQUAL LIVES

Incremental Analysis			Present Value Factor at 12%	Present Value Amount
If Machine B is purchased now:				
Annual operating savings ($10,000 − $3,000) (Note 1)	$7,000	$ 7,000		
Additional depreciation ($2,000 − $1,000) (Note 1)	1,000			
Taxable saving	$ 6,000			
Tax at 48%		2,880		
Post-tax saving (Note 2)		$ 4,120	3.037	$12,513
At end of 4 years: Disposal value of Machine B (no gain or loss on book value) (Note 3)		$12,000		
Less: Disposal value foregone of Machine A (Note 4)	$ 1,000			
Capital gain tax at 25%	250	750		
		$11,250	.636	7,155
Proceeds on disposal of Machine A realized now		$ 2,500	1.0	2,500
Loss on book value of Machine A ($4,000 − $2,500)	$ 1,500			
Tax saved at 48%		$ 720	1.0	720
Total of all cash inflows				$22,888
Investment in Machine B (cash outflow)		$20,000	1.0	20,000
Excess present value (in favor of replacement now)				$ 2,888

Notes on Solution:

Note 1: Only the difference in operating costs is considered. For tax impact, only the additional or incremental depreciation between Machines A and B is taken into account.

Note 2: The same result could be obtained as follows:

Annual operating savings	$7,000
Additional depreciation	1,000
Taxable savings	$6,000
Tax at 48%	2,880
Balance	$3,120
Add back depreciation	1,000
Post-tax saving	$4,120

Note 3: Since the remaining life of Machine A is 4 years, it is necessary, for comparative purposes, to estimate the residual value of Machine B at the end of 4 years.

Note 4: If Machine B is purchased now, the company loses the opportunity to realize $1,000 4 years from now. This is what economists call an **opportunity cost.**

Source: Bogen, *op. cit.*, p. 17.45

The above example permits a visible corroboration of the following equation derivations.[1]

(1) After-tax proceeds = revenues-expenses other than depreciation-income tax

(2) Since: income tax = (tax rate) x (taxable income) and depreciation is tax deductible

(3) Income tax = (tax rate) x (revenues-expenses other than depreciation-depreciation)

Substituting equation (3) in equation (1) gives equation

(4) After-tax proceeds = (1-tax rate) x (revenues-expenses other than depreciation-depreciation) +(depreciation)

or

(5) After-tax proceeds = (1-tax rate) x (revenues-expenses other than depreciation) + (tax rate) x depreciation

The Internal Revenue Code of 1954 allows a firm a choice of depreciation methods. In the above example, straight-line depreciation was used for simplicity of illustration. However, since the choice of depreciation method will affect investment profitability, it behooves a firm to explore the method that will maximize its profits. The method chosen may depend upon the firm's cost of capital as well as upon the life of the investment and its expected salvage value. However, it is rare that a firm will depreciate equipment with

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[1]Bierman and Smidt, *op. cit.*, pp. 104-105.

other than an accelerated method. The underlying impetus is more often than not the threat of functional obsolescence. Long-lived assets such as a utility distribution system are commonly depreciated by the straight-line method.

Equation (5) is particularly adapted to calculating the optimum method of depreciation when used in conjunction with the present-value method. The first part of the equation is independent of the depreciation method, the second fully dependent.

Table 14 shows the use of the latter part of equation (5) as well as the methodology of the three depreciation measures. Since the initial part of the equation is the same for either method, the one producing the highest present value in the table also produces the highest present value overall.

Some firms, particularly manufacturing companies, treat similar assets as a category. If the assets are alike, such as portable welders, with approximately the same useful life, the handling of such equipment is termed group depreciation. If unlike assets such as office equipment are treated together, the process is labeled composite depreciation.[1]

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[1] Anthony, *op. cit.*, p. 158.

PRESENT VALUE OF TAX SAVINGS FROM DIFFERENT
METHODS OF DEPRECIATION (10% DISCOUNT)

	DEPRECIATION METHOD								
	Straight Line			Twice Straight Line			Sum-of-the-Years' Digits		
YEAR	Allow. Expense	Savings (52%)	Present Value	Allow. Expense	Savings (52%)	Present Value	Allow. Expense	Savings (52%)	Present Value
1	$2,500	$1,300	$1,182	$5,000	$2,600	$2,364	$4,000	$2,080	$1891
2	2,500	1,300	1,074	2,500	1,300	1,074	3,000	1,560	1289
3	2,500	1,300	977	1,250	650	488	2,000	1,040	781
4	2,500	1,300	888	1,250	650	445	1,000	520	356
T	$10,000	$5,200	$4,121	$10,000	$5,200	$4,371	$10,000	$5,200	$4,317
EXAMPLE	$\frac{10,000}{4 \text{ years}}$ = $2500/yr.			yr. 1 10,000x1/2=5000 yr. 2 5000x1/2 =2500 yr. 3 2500x1/2 =1250 yr. 4 1250x1* =1250 *Asset retired, full balance applied.			yr. 1 10,000x4/(4+3+2+1) =10,000x4/10=4000 yr. 2 10,000x3/10=3000 yr. 3 10,000x2/10=2000 yr. 4 10,000x1/10=1000		

Source: Adaptation of Table in Bierman and Smidt, *op. cit.*, p. 110.

Under the group method, the rate of depreciation is based on the average life of many similar units. This rate is then applied to the balance of units in use.[1] The depreciation rate in composite depreciation is a weighted average rate, the weighting reflecting the dollar amounts of assets in each of the various categories involved. Thus, a single depreciation rate is derived for all office equipment in a plant.[2] The annual depreciation expense under group or composite depreciation depends on the method used. In the case of straight-line application, the rate is applied to the total cost of the asset group. When the group or composite methods are used, no gain or loss is recognized when an individual asset is disposed of. This procedure assumes that gains on some dispositions are offset by losses on others.[3]

Salvage value affects each yearly computation of the sum-of-the-years' digits, but by using the double-declining-balance method, this problem can be avoided while still benefiting from accelerated depreciation.[4]

To simplify the inherent arithmetic in depreciation computations, most large firms use tables to assist in the

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[1]Bierman and Smidt, *op. cit.*, p. 113.

[2]Anthony, *op. cit.*, p. 159.

[3]*Ibid.*

[4]Bierman and Smidt, *op. cit.* p. 113.

choice of a method and to derive the tax savings associated with an investment proposal.

While the ramifications of depreciation planning under the Internal Revenue Code would in itself provide material enough for a doctoral dissertation, the subject has been confined in this paper to some of the more important complications that arise in practice.

Inflation and Investment Selection

In an age of seemingly irrepressible inflation, an investment analyst might well ask how the prospect of inflation will affect his analysis of investment proposals. Three questions bear on this problem:

(1) Will the projected net cash flows respond to inflation?

(2) When the dollar is changing in absolute buying power, how can the rate of return be computed?

(3) Are taxes due when receipts are received or later?

The latter question is vital and requires a conditional answer. If the tax laws permit payment of taxes in year 2 for sizable cash inflows projected for year 1, there is valid consideration in favor of the project. If the margin between "go" and "no-go" for the investment rests on this basis

as a *premise*, the investment should be rejected unless confirmation of delayed payment is available.[1]

The answer to question (1) is that receipts predetermined by contract, such as a bond, do not respond to inflation. In the case of physical production assets, the receipts can be expected to be fully responsive.[2]

As for the nebulous return on investment during times of inflation, Terborgh states, "*No rational computation is possible without converting future receipts in varying dollars into their equivalent in the dollars of investment or some other uniform measure.*[3]

SUMMARY

The preceding chapter was devoted to an overview of methods used (and misused) for the measurement of investment worth. The resultant of this inquiry was that the value of any method (while perhaps technically superior to others, or simply adequate for the task at hand) was dependent upon the relevancy and accuracy of the data applied to it.

In this connection, Chapter III was designed to ferret out those concepts and quantitative inputs intrinsic

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[1]Bierman and Smidt, *op. cit.*, p. 119.

[2]Terborgh, *op. cit.*, p. 191.

[3]*Ibid.*, p. 192.

to a comprehensive and methodologically correct cash-flow analysis. This approach perhaps reflected a bias toward the discounted cash-flow methods but, nonetheless, is *a priori* of investment analysis and, resultantly, selection.

Having noted that good data is synonomous with effective cash-flow application, it is also apparent that cash-flow composition takes place in a milieu of varying degrees of certainty and acknowledged risk. Since risk and uncertainty are apparently concomitants to the estimation of future cash flows, they are also factors germane to the investment-selection process. As such, they are examined in Chapter IV.

CHAPTER IV

RISK AND UNCERTAINTY

An inherent and omnipresent aspect of capital budgeting is decision-making, where choices must be made on alternative courses of action. The quantitative data used in these decisions are normally estimates of outcomes for each alternative course of action. However, estimates are forecasts, they relate to the future and, therefore, embrace the uncertainties association with forecasting.[1] And, yet, "...uncertainty characteristics are important when there is much obsolescence of methods or of style, or fickleness and obscurity of forecasting buyers' tastes, particularly in development of new and unknown product lines."[2]

Considerable emphasis has been given the application of probability techniques to measure or otherwise provide for risk and uncertainty in estimates. The need for this effort is succinctly advanced by Joel Dean.[3]

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[1]Bogen, *op. cit.*, p. 17.75.

[2]Joel Dean, *Managerial Economics*, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1951), p. 568.

[3]*Ibid.*

> *Adjustments to allow for uncertainty may be challenged as nothing more than guesses. Perhaps they are. But even so, they are guesses that must be made, and will be made, either explicitly or implicitly. Failure to apply the probability adjustments does not enable management to avoid the problem: it merely transfers the quess element in disguised form to some other stage of the decision-making process.*[1]

Definition and Sources

This recognition and research of risk and uncertainty has for some analysts produced a dual concept of the subject, rather than simply synonymous terminology--i.e., risk and uncertainty are different, though integral to the same problem area.

Spencer and Siegelman define risk as "...the quantitative measurement of an outcome, such as a loss or a gain, in a manner such that the probability of the outcome can be predicted."[2] In other words, risk is objective and quantifiable, in that its probability can be measured with certainty where the outcome characteristics are known in advance, as in coin-tossing or dice-throwing.[3]

Another application is that of prediction based on prior experience or historical data, where the number of

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[1] *Ibid.*

[2] Milton H. Spencer and Louis Siegelman, *Managerial Economics-Decision-Making and Forward Planning,* (Honewood, Ill.: Richard D. Irwin, Inc., 1959), p. 5.

[3] Bogen, *op. cit.,* p. 17.75.

observations or data are large enough to exhibit stability, are repeated in the population, and are independent (or randomly distributed). The prediction is ordinarily influenced by the normal frequency distribution whose characteristics can be defined by measuring (computing) the mean (central tendency), standard deviation (dispersion), and skewness.[1]

Shubin describes uncertainty as that factor that "...arises from developments and fluctuations that cannot be sufficiently foreseen and adequately predicted, such as changes in consumer style tastes and unforeseeable product developments by rival firms. Technological advancement also increases uncertainty when it renders obsolete certain productive facilities."[2]

In contrast to risk, uncertainty is subjective in nature. While the data concerns the future, it cannot be objectively verified, as in games of chance. Thus, different persons may estimate a given outcome quite differently, on nothing more than intrinsic optimism or pessimism.

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[1] *Ibid.*

[2] John A. Shubin, *Managerial and Industrial Economics* (New York: The Ronald Process Co., 1961), p. 356.

This paper will address the objective side of risk and uncertainty which, by the above definitions, are essentially the analysis of and coping with risk. However, this writer feels that the two concepts are not clearly discernible as to the individual entity, that they do overlap, and hence the title of the chapter includes both terms.

Merrett and Sykes track down risk in capital budgeting decisions as generally emanating from five sources:[1]

(1) *Risk from insufficient numbers* (of similar investments) arises when the firm undertakes an investment with which it has had little or no prior experience. Unfamiliarity may lead to erroneous estimates which by themselves may produce a bad decision. However, the law of averages also enters the analysis. All things being equal and accurate, like a toss of a fair coin, the project may fail to materialize simply because of failure of the law of large numbers to operate with a single trial of the probability mechanism.

(2) *Risk from external change* results from changes in the probability mechanism. A mechanism containing a change prognosis would be impossibly complex. Some attempt to incorporate the more likely and discernible change factors may be

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[1]Merrett and Sykes, *op. cit.*, pp. 180-183.

made but generall such changes are the result of external and larger, more complex forces.

(3) *Risk from misinterpretation* of the complex probability mechanism is common. For instance, a sales forecast not only entails an understanding of the nature and magnitude of each forecast variable, but also the primacy of the relationships among the variables (and constants). In the present state of knowledge concerning the latter, great latitude for error in forecasting exists, and such error is often the main element of risk involved in a capital project.

(4) *Risk of bias* frequently arises anytime there are divergent opinions, or multiplicity of attitudes. Bias can enter an evaluation very subtly; and, depending upon the hierarchial level of source, bias can have a very significant bearing on the decision process.

(5) *Risk from errors of analysis* primarily occurs in the technical and financial analysis of the project. This is particularly common when the technical process being used is unusual, or where the project implications are complex and numerous. These risks come from errors of commission as well as omission.

Before entering the arena of risk analysis, the practice of investment classification should be mentioned. For administrative purposes, most enterprises first assemble preferred new investments and then group them into categories.[1] This procedure recognizes that the accuracy of cash-flow estimates varies among types of investments by reasons of complexity and uncertainty.[2] Replacement or cost reduction investments are generally characterized by low-marketing risk and a relatively straightforward analysis, while expansion/diversification (new product) undertakings carry with them a moderate to high degree of risk.[3] While the classification process varies greatly among firms, it is widely recognized as a fundamental step in risk analysis.

Recognition of risk by degree and nature is tacit acknowledgement of the fact that sound intuitive judgment, while a necessary part of a sound decision is not sufficient to guarantee it. Analysis is required to ensure that no essential factors have been omitted from the basic premises and that each factor is assigned an appropriate weight.

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[1] J. Fred Weston and Eugene F. Brigham, *Essentials of Managerial Finance* (New York: Hold Rinehard and Winston, Inc., 1968), p.128.

[2] Dean, *op. cit.*, p. 82.

[3] Weston and Brigham, *op. cit.*, p. 128.

"Analysis is no substitute for sound intuitive judgment, but neither is such judgment a substitute for analysis."[1]

Risk Analysis

There are several limited methods now in use that allow for the presence of uncertainty so that the analysis may proceed. However, since all but one (more accurate forecasts) employ subjective considerations, discussion of these efforts will be limited to a single example.

One way to improve the quality of cash-flow estimates is to use empirical data to adjust the estimates.[2] If, for example, the firm's history of equipment costs is such that improvements and obsolescence have exceeded estimates by 10 percent, then this chronic error is justifiably compensated for in the new project estimates.

A more subjective process often assails a project as it moves up through the firm's hierarchy. Despite the quality of the analytical analysis that produced the benefit figures, arbitrary "hedge" cuts in these projected benefits are often made. This procedure can lead to rejection of the project at the highest levels of authorization by a management

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[1]Merrett and Sykes, *op. cit.*, p. 178.

[2]Hertz, *op. cit.*, p. 98.

ignorant of the true potential value of the investment. This might be tabbed "bureaucratic risk". The recommended cure is to make the analyst less liable for his estimates and attach less significance to performances that exceed highly conservative expectations.[1]

Weighted Multiple Estimates

For any input in an estimate, there is normally a subjective but recognizable range of possibilities, with a pessimistic alternative at the lower limit and an optimistic alternative as the upper limit. Working with a wide spectrum of estimates may be unfeasible. A widely used solution is to assign a weight to the pessimistic, most probable, and optimistic estimates. (The weight factors are probabilities, and as such add up to unity). The product of the estimate and its assigned weight gives the expected value. The averaged sum of the expected values represents the weighted expected value for the input being considered. This derived value to some degree expresses the uncertainty of the input.

An expanded version of this method utilizes all significant values in the spread of values for each factor. A relative probability representing the liklihood of occurrence

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[1] Donald H. Woods, *"Improving Estimates that Involve Uncertainty," Harvard Business Review, XLIV, No. 4* (July-August, 1966), p. 96.

is assigned each value. This procedure is repeated for each factor that enters into the equation for which an answer is sought. A particular value is selected at random from each distribution of values for each factor. These selections are then combined to solve the equation (normally the rate of return or present value of cash flows).

Through the use of a computer this procedure can be repeated such that every possible combination is run through. The results will be a range of values from loss to maximum gain. The mathematical model has thus yielded all possible values. Since a given value can be achieved through one or more combinations, the higher the number of combinations the greater the chances of achieving that particular value. The average expectation is the average of the values of all outcomes as weighted by the likelihood of occurrence. The result is depicted in Figure 5.

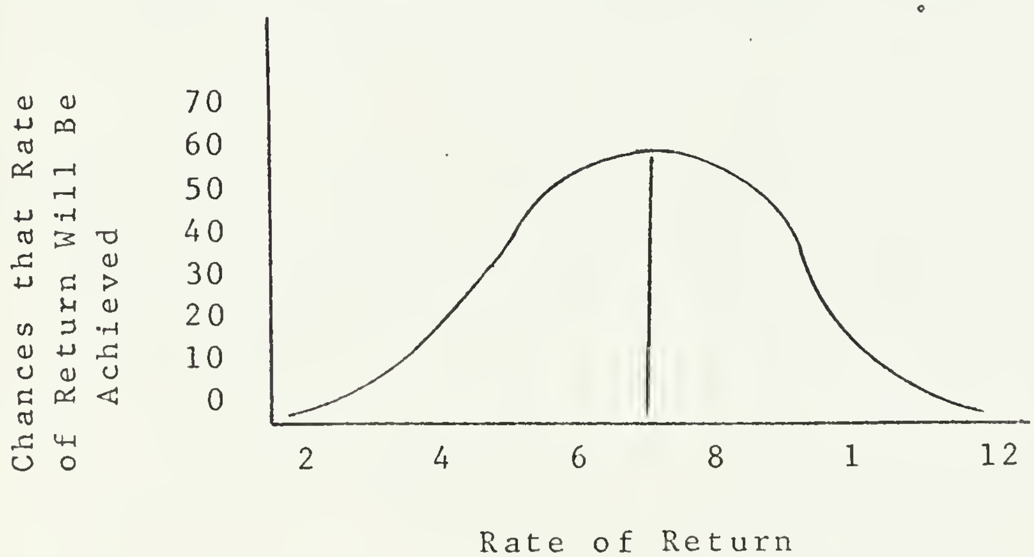

Figure 5. PORTRAYAL OF INVESTMENT RISK WITH RATE PLOTTED BY CHANCES OF OCCURRENCE

Source: Hertz, *"Capital Investment"* p. 102.

The variability of outcome values from the average is also important since rational management would prefer lower variability for the same return if the choice were available.[1]

The value of this approach (and its superiority over rules-of-thumb or the limited-variable-input method previously discussed) is that it considers a range of values for each significant factor. The ranges normally reflect a composite viewpoint, often reinforced with historical data or trends. The weight assigned each factor, as a relative probability value, normally represents the opinion of the firm's most knowledgeable individual(s) in that respective area. Furthermore, it poses no special problem to any firm with access to a digital computer. However, its most striking advantage may be that it allows management to discriminate between measures of:[2]

(1) expected return based on weighted probabilities of all possible returns;

(2) variability of return; and

(3) risks.

This can be illustrated in figure 6 where:

-- Investment B has a higher expected return (6.8%) than investment A (5.0%) at a 50% chance of achievement.

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[1]Hertz, *op. cit.*, pp. 99-100

[2]*Ibid.*, p. 105.

-- Investment B has a variability ranging from a loss of 10% to a positive 25% as shown on Figure 6. Investment A has very little variability.

-- Investment B entails considerably more risk than A, again as indicated by the relative plots of the two investments.

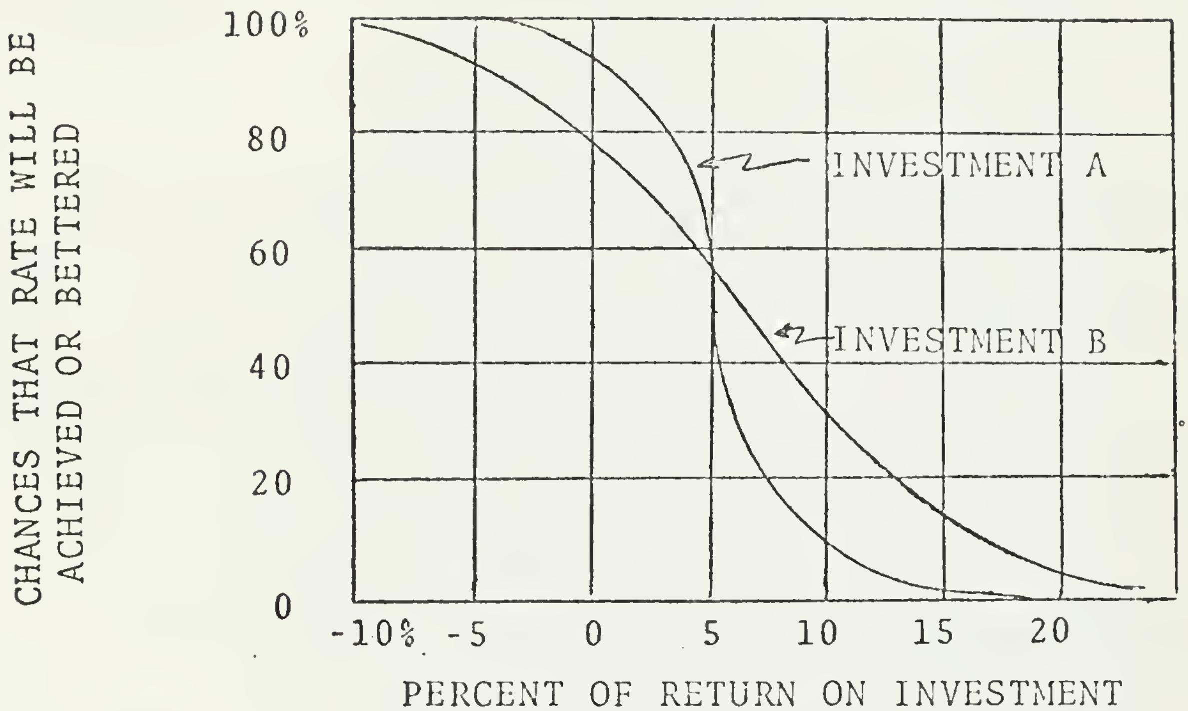

Figure 6. COMPARISON OF RATES OF RETURN FOR TWO INVESTMENTS BY CHANCES OF OCCURRENCE

Source: Hertz, "*Capital Investment*," p. 105.

This method is a first-rate management tool since it necessitates finding out a great deal about the key factors in the problem and the incorporation of the most authoritative opinion available. Hence, the kind of uncertainty that is involved in each estimate can be evaluated ahead of time, allowing management to maximize the value of the information for decision-making.[1]

Décision Trees

The decision-tree approach is a way of displaying the anatomy of a business decision. It enables management to take more direct account of:[2]

-- the impact of possible future decisions;

-- the impact of uncertainty;

-- the relative value of present and future profits.

A detailed exploration of this concept is beyond the intended scope of this paper. In portraying its use in risk analysis, a limited application will be shown. The following is an adaptation of one used by Edward J. Mock, et al., *(Basic Financial Management)*.[3]

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[1] *Ibid.*

[2] John F. Magee, *"How to Use Decision Trees in Capital Investment," Financial Decision-Making*, ed. by Edward J. Mock (Scranton, Pa.: International Textbook Company, 1967), p. 471.

[3] Mock, *op. cit.*, pp. 184-186.

The company has a cost of capital of 10 percent. It is considering an investment proposal of $100,000 that has an expected economic life of three years. After-tax proceeds are projected as follows:

End of year	Dollars	Assumed Probability
1	$150,000	.6
	80,000	.4
2	200,000	.7
	0	.3
3	100,000	.8
	70,000	.2

The problem of outcomes will be studied under two cases: Table 15 where all events are statistically independent, and Table 16 where some events are statistically dependent.

In the first case, this would be a "must" project with a net present value of $197,200. There is virtually no risk, since even the "worst" possibility exceeds zero, thereby allowing recapture of the investment at an interest rate slightly above the cost of capital.

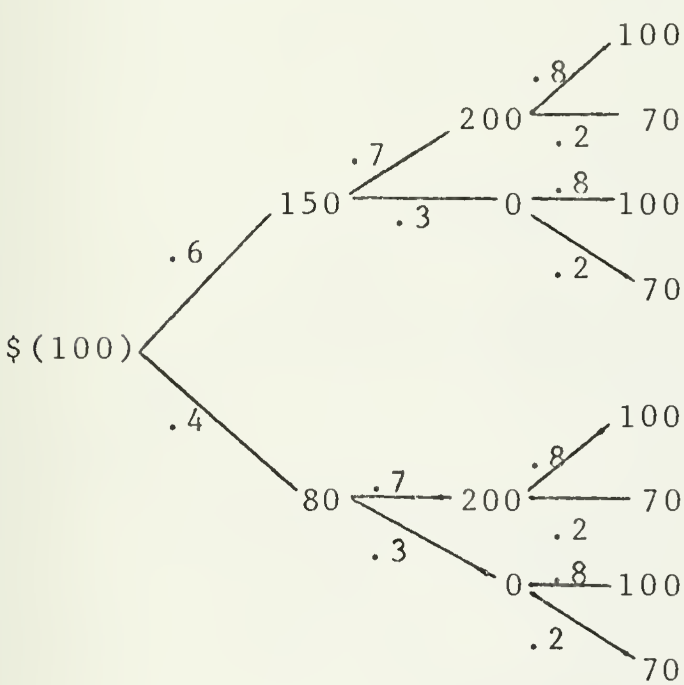
However, in the second case, the amounts in year 3 are statistically dependent on one-half the amounts received in year 2 and the amounts in year 2 are statistically independent of year 1.

Table 15

ANALYSIS OF HYPOTHETICAL INVESTMENT ALTERNATIVES USING A DECISION TREE WITH NET PRESENT VALUE METHOD

[All events are statistically independent. All cash flows in thousands]

Year 0	Year 1	Year 2	Year 3



Net Present Value of Path	Probability of Path
$276 [1]	.336 [2]
253	.084
111	.144
88	.036
213	.224
190	.056
48	.096
25	.024

[1] Present value of path equals cash flow for year x $\frac{1}{(1+i)^n}$ where n = year and i = .10:

150 x .909 + 200 x .826 + 100 x .751-100 x 1.0 =
136 + 165 + 75-100 = 276

[2] Probability of path equals probability of year 1 x probability of year 2 x probability of year 3: .6 x .7 x .8 = .336

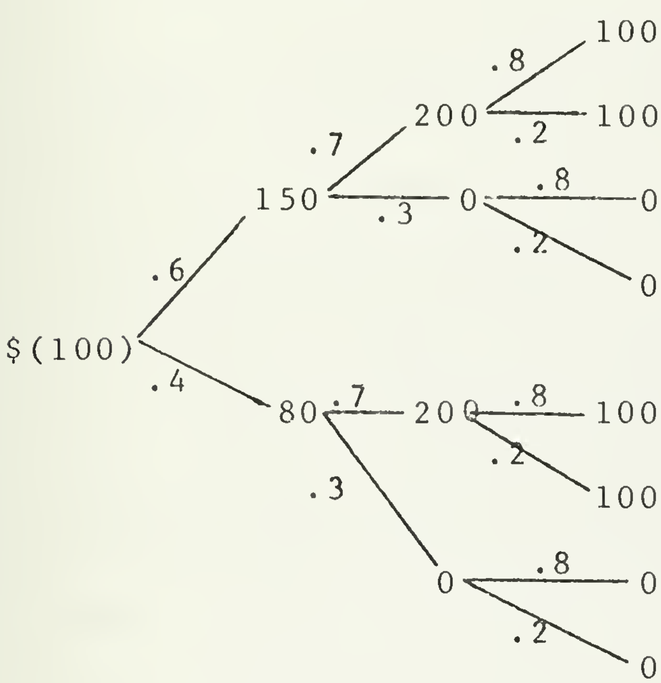
Probability	x	Value: Conditional	=	Value: Expected±
.336		$276,000		$ 93,000
.084		253,000		21,000
.144		111,000		16,000
.036		88,000		3,200
.224		213,000		48,000
.056		190,000		10,600
.096		48,000		4,600
.024		25,000		500
Expected net present value				$197,200

Table 16

ANALYSIS OF HYPOTHETICAL INVESTMENT ALTERNATIVES
USING A DECISION TREE WITH NET PRESENT VALUE METHOD

[Events are partially dependent statistically, vice independent. See page 102. (Cash flow in thousands.)]

Year: 0 1 2 3



Net Present Value of path±	Probability of Path
$276	.336
276	.084
(100)	.144
(100)	.036
213	.224
213	.056
(100)	.096
(100)	.024

Probability	x	Conditional Value	=	Expected Value
.42		276,000	=	116,000
.28		213,000	=	59,700
.30		(100,000)	=	(30,000)
Expected net present value				.145,700

This project is also lucrative from the standpoint of an NPV of $145,700. However, there is a 30 percent probability of a $30,000 loss. This might well deter a firm with marginal capital resources--particularly if it has a relatively high-debt-to-equity ratio or problems with market valuation of its stock.

This decision tree provides easily comprehended schemes of outcomes for various investment conditions; outcomes that recognize risk in terms of subjective probabilities and statistical analysis. But what about the situation where a decision triggers only one or two outcome branches, which in turn lead to a second decision point? This is a different situation that the start-to-finish chain reaction previously demonstrated. It employs the element of time in the decision scenario, along with the customary "tree" of alternatives and associated probabilities, costs and returns.

Referring to Figure 7, at Decision Point 1, the company must decide between alternative A and B. However, depending on the outcome of B, a second decision may be necessary. This is indicated by Decision Point 2, which, in the decision tree, follows Decision Point 1 by two years. This second decision produces additional possibilities which are a part of the total

analysis. It may be necessary to first analyze Decision Point 2 to properly evaluate the significance of the B alternative from point 1.

Decision trees, in portraying the information available to management, make possible more systematic analysis and lead to better decisions. To use this analytical tool management must:[1]

1. Identify the points of decisions and alternatives at each point.

2. Identify the points of uncertainty and the type or range of alternative outcomes at each point.

3. Estimate the values needed to make the analysis, especially the probabilities of different events or results of action.

4. Analyze the alternative values to choose a course.

A decision tree does not give management investment decisions. Rather, it provides assistance in determining the super alternative at any decision point. The unique feature of the decision tree is that it allows management to combine analytical techniques such as discounted cash flow (yield) and present-value methods with a relatively clear portrayal of the impact of future decision alternatives and events.[2]

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[1] John F. Magee, *"Decision Trees for Decision-Making," Harvard Business Review, 42* (July-August, 1964), p. 133.

[2] *Ibid.*, p. 138.

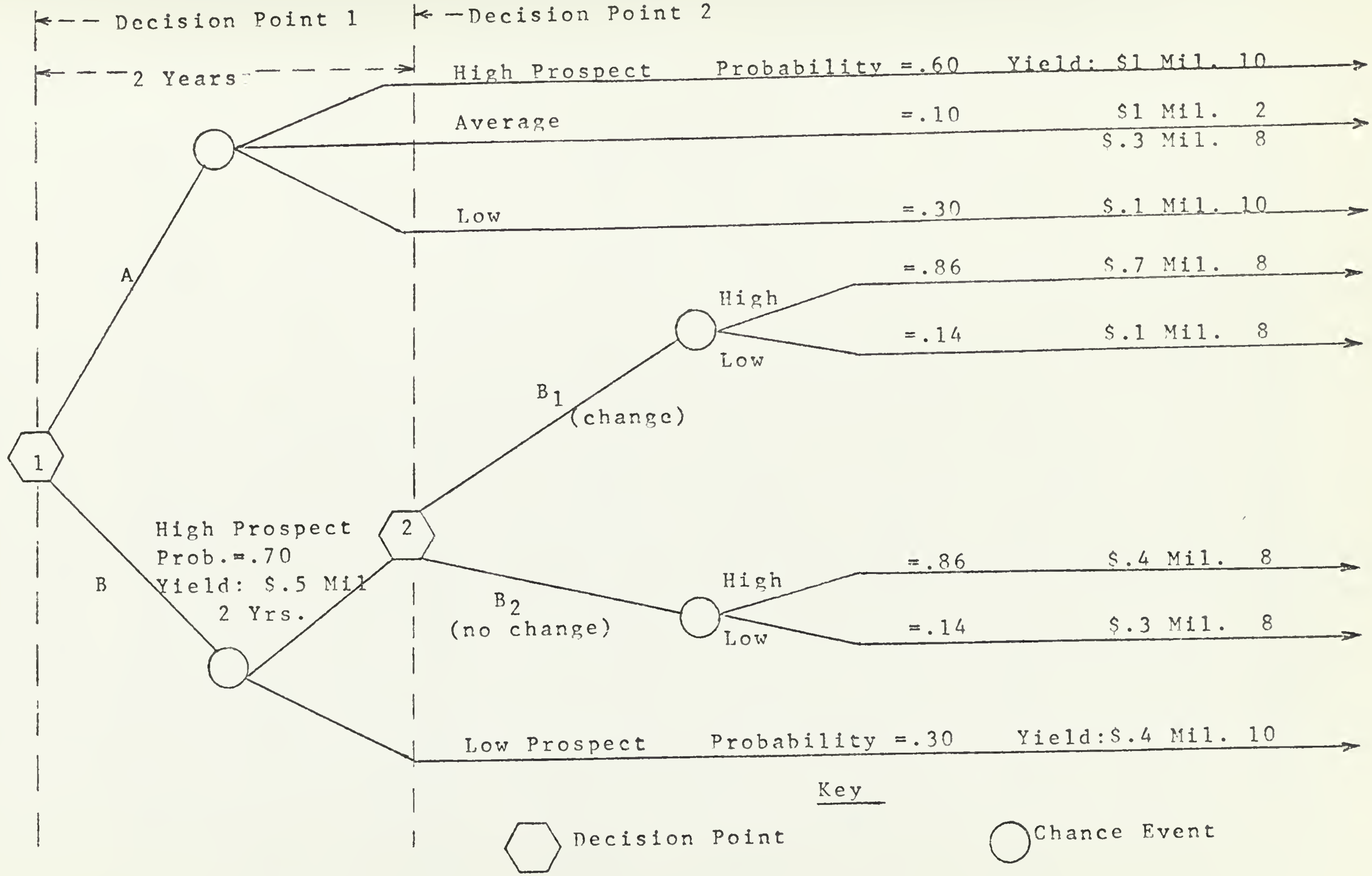

Figure 7 Decision Tree with Multiple Decision Points

Source: John F. Magee, "Decision Trees for Decision Making," *Harvard Business Review*, 42 (July-August, 1964), p. 132.

Return on Investment Versus Cost of Capital

The next chapter is devoted to the cost of capital with the purpose of rounding out the financial determinants involved with investment selection. This cost of capital to a firm has many interpretations both in computational method and significance, depending on the "authoritative" source consulted. It any event, the yield or present-value discount rate of a proposed project can be compared to the firm's cost of capital. If the latter is computed correctly, the investment yield should exceed (or at least equal) the cost of capital; or the rate of interest used in the present-value equation should be no less than the cost of capital. To do otherwise would mean funding a project that has a projected rate of return less than the cost of funds invested in it.[1]

When a project return is compared with the cost of capital, the question often provoked is: What should be the spread between the cost of capital and the yield of an acceptable project, so that risk is properly recognized? This question can also apply to the net present-value method in the form of an added percentage to the cost of capital for use in the NPV equation.

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[1]Pollack, *op. cit.*, p. 11.

Adding an arbitrary percentage to the cost of capital to compensate for risk "merely transfers the guess element in disguised form to some other stage of the decision-making process."[1] It would be far more rational to just apply probability adjustments to cash-flow projections.[2] Adding a percentage to the discount rate in the NPV calculation is also technically unsound, for the risk originally assigned in this fashion is discounted by its own original value for each year of application.[3] Adding a "risk factor" of .05 to a cost of capital of .10 and then discounting back at .15 is equivalent to stating that:

$$\frac{1}{(1+.1)^n} + \frac{1}{(1+.05)^n} = \frac{1}{(1+.15)^n}$$

Which is erroneous.[4]

An aspect of risk that may escape the analyst (particularly one preoccupied with arbitrary adjustments for investment business risk) is that portion of the market interest rate representing the lenders' recognition of risk. As previously mentioned, the rate of interest demanded by the lender is not just

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[1]Dean, *Managerial Economics*, p. 568.

[2]Bierman and Smidt, *op. cit.*, p. 128.

[3]Refer to Chapter II for further discussion on this point.

[4]Edward R. Oscarson, *"Capital Investment Analysis in the Navy"* (Unpublished Master's Thesis, George Washington Univ., 1967), p. 38.

a reflection of his disutility in postponing consumption, but also represents compensation for the nature of the risk he associates with the use of his money by others.[1] This lender-risk-consciousness is not necessarily all fear of outright loss, such as through failure of the investment, but may also reflect aversion to high opportunity costs--i.e., having tied up funds in a low-return (but relatively safe) investment, subsequent higher return opportunities will have to be forgiven. This aspect has implications in the cost of funds and is further discussed in Chapter V, Cost of Capital. For purposes of this chapter, it is re-emphasized that safety and high-profit opportunity can normally be found at opposite ends of the investment spectrum; and that the price paid for funds used in the investment scenario includes compensation to the lender for the risk taken in making the funds available.

Prospect

While risk analysis and coping with uncertainty will no longer be regarded by the business community as the "edge of the world," it is unlikely to become familiar ground to small and intermediate-size business--at least not in the immediate future. This speculation is based on the very theme of the paper--the quantitative aspect.

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[1]See page 14, Time Adjusted Methods.

To appreciate the subjective implications of risk, and uncertainty, the successful businessman has but to look into his past. However, to quantitatively evaluate risk in even the academic neatness of the above examples is to possess working knowledge of certain analytical techniques. Effectively projecting this endeavor into the maelstrom of the major business decision would require familiarization with operations research methodology that is not likely found in the great majority of medium- and small-size enterprises.

It may be a matter of marketing. Perhaps what is needed is the same sort of enterprise impetus that launched the computer so quickly and solidly into prominence. A nimble-minded entrepreneur with some of the alleged current surplus of highly educated talent may change this state of affairs. Investment consultants specializing in risk and uncertainty could conceivably make probabilities as common as net present value (although the latter is over twenty years old and still a minority technique). Like most products, the costs would have to be some fraction of the benefits. Deriving the latter would be akin to compounding uncertainty. However, just as the family farm is fast becoming an anachronism, so may business rules-of-thumb. The struggle of competition will continue as a way of business life, and risk evaluation is a (double-edged) weapon.

SUMMARY

There are a variety of techniques for incorporating risk (and uncertainty) into the various facets of capital investment analysis. The use of probabilities is becoming more widely accepted in the larger business entities where capital budgeting is recognized as a cardinal function and is staffed appropriately. (Recognition is not uniform, however, and is particularly tenuous in the small business realm.) The concomitant emergence of the computer has obviously enhanced this adoption. Also on the ascendency appears to be a general recognition of risk and a basic understanding of its characteristics--if the relatively recent volume and diversity of writings on the subject is a valid indicator. Certain mathematical tools such as sequential decision theory, network analysis, Markov chain analysis and linear programming are devices to refine such variables as the value and direction of decision alternatives, optimum timing, and maximum/minimum factor combinations--all of which mitigate the risk in estimates.[1]

Uncertainty in connection with large losses may force management to consider tools to measure the importance attached to possible large losses versus large profits. The question of

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[1]Wasson, *op. cit.*, pp. 181-218, *passim*.

psychological impact on managers and investors alike complicates such analysis.[1] While this area of analysis is still in its infancy, the terminology such as linear and nonlinear utility, expected loss, and discrete and continuous events is being used in connection with models to determine such things as personal evaluation of risk and time preference.[2]

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[1]Bierman and Smidt., *op. cit.*, p. 132.

[2]Harold Bierman, Jr., Charles P. Bonini, Lawrence F. Fouraker and Robert K. Jaedicke, *Quantitative Analysis for Business Decisions*, 1st rev. ed. (Homewood, Illinois: Richard D. Irwin, Inc., 1965), pp. 354-375.

CHAPTER V

COST OF CAPITAL

According to Mock, et al., "No capital investment should be undertaken unless its expected discounted cash-flow rate of return (computed from anticipated cash outlays and inflows) exceeds the anticipated cost of capital by some factor."[1] From a monetary standpoint, an alternative decision rule can be formulated. The present value of the expected stream of net incremental cash inflows generated by an investment must be greater than the present value of the anticipated net cash outlays required by the investment when both inflows and outlays are discounted at the cost of capital.[2]

Despite its implication in the investment decision, the cost of capital is not a precise figure, to be computed with finality. If the market price of a company's common shares changes, it precipitates a change in the firm's cost of capital.[3] Similarly, a change in the funds markets will affect the company's capital cost that has (or intends to use) debt capital. However, though it is difficult to predict a

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[1]Mock, Schultz, Schultz & Shuckett, *op. cit.*, p. 203.

[2]*Ibid.*

[3]Bogen, *op. cit.*, p. 17.48.

firm's cost of capital with precision, it can and must be estimated[1]--not calculated as a single figure, or cutoff point, but rather as a value in a boundary area.

As R. W. Johnson points out: "Designating a cost of capital at a certain percentage for a given extent of financing really means that as the rate of return after taxes on capital expenditures approaches the designated percentage, the examination of proposed projects should be increasingly rigorous."[2]

A comprehensive treatment of the determination of the cost of capital is beyond the purview of this study, since the intent of this portion of the paper is merely to demonstrate the relationship between cost of capital and the discount rate used in investment analysis. Accordingly, the scope of this chapter will be limited to an introduction of the concept of the cost of capital, and a curtailed analysis of methods for deriving and applying it. The implications of capital rationing will be disregarded. Though it is present throughout the economy in one form or another, discussion here would serve no purpose.

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[1]John F. Childs, *Long-Term Financing* Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961), p. 324.

[2]Johnson, *op. cit.*, p. 17.

The Concept of the Cost of Capital

A primary responsibility of a firm's management is to maximize the net present worth of the residual owners. In this capacity, the cost of obtaining capital for the firm's operations is also the minimum required rate of return on any investment demanding such funds.[1] Thus, in theory, a firm could be expected to obtain funds and invest them in capital projects as long as the discounted rate of return exceeds or, as a minimum, equates to the cost of capital.[2] However, this theoretical concept is influenced by a number of factors, which, responding to the expansive actions of the firm, would divulge a series of costs of capital, each one higher than its predecessor. This characteristic stems from the fact that these funds must be bid away from other competitive demands in the funds market. Parenthetically, as a firm moves down through its list of prospective investments, (arranged in descending order of profitability) the aspect of risk and uncertainty would be expected to interpose a barrier at some point to any further undertakings. But, even without the prospect of risk commensurate with utilization, the acquisition

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[1]Mock, Schultz, Schultz and Suckett, *op. cit.*, p. 205.

[2]Bogen, *op. cit.*, p. 17.48.

of additional capital is not immune to the classic laws of supply and demand, whereby unit cost increases with unit demand. Reduced marginal utility can be expected as successive issues of stock or bonds result in proportionally less net proceds. This situation will normally continue until earnings improve.

This writer feels that risk plays a significant part in the supply-demand scenario. While, human nature certainly exerts itself in the higher costs that follow increased demand and limited supply, the *nature* of the increased *demand* is also pertinent. An expanding economy breeds speculative as well as astute and conservative investment. Often the former increases at a faster rate than any other type. Lenders will tend to price their money in consonance with the overall risk they feel they are taking. On one hand is the risk of loss of their capital, and on the other is fear of losing potentially higher earnings on future opportunities if they tie-up their money in safer but lower return investments. It is not implied here that the risk of opportunity losses (opportunity costs) will at some point equate to risk of financial losses (through business failures), but rather that one tends to influence the other. The point to be made is that the interest rate (cost of money)

partially represents compensation for risk; and that the nature and, hence, extent of this risk is influenced by the borrower's financial condition as well as the current trend of the market. The resultant rate will favor the firm with the superior risk status, while the less stable enterprise will find its less attractive financial structure "supporting" a higher cost of funds. In either case, the financial status being evaluated by the lender would presumably include the influence of the business risk associated with the investment for which funds are sought.

At this point, it might be well to mention that the traditional concept of capital structure would have the average cost of capital decreasing as debt is introduced.[1] However, after a certain point is reached, the average cost would rise as stock and bond yields are forced up because of the growing financial risk represented by the increasing debt. The implication here is that increased debt eventually dilutes the quality of a stock, and that greater compensation in the form of higher yield is necessary to satisfy the investor. This theory is widely held, though apparently not uniformly conceived, nor expressed.

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[1]James C. T. Mao, *Quantitative Analysis of Financial Decisions* (London: Collier-Macmillan, Ltd., 1969), p. 416.

Various authors have proposed an assortment of models, some of which are more dynamic (and comprehensive) than others.[1] In 1958, Modigliani and Miller presented a new theory in financial management.[2] The substance of their work lay in the proposition that in a world of perfect markets and rational investors, two identical companies with similar assets and earnings of like quality must have the same total market value, regardless of differences in leverage. Based on this proposition, market value and hence cost of capital are independent of capital structure.[3] This treatise was promptly attacked by a number of scholars in the field of financial management.

Probably the best known rebuttal to Modigliani and Miller is by David Durand, who acknowledges the validity of the MM propositions but stresses their unrealistic and "extremely restrictive nature." Durand concentrates his counter argument on the MM proposition delineated above, which

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[1] Alexander Barges, The Effect of Capital Structure on the Cost of Capital (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963),

[2] Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment", American Economic Review, 48 (June, 1958), pp. 261-296, passim.

[3] Edward J. Mock, Financial Decision-Making (Scranton, Pa.: International Textbook Company, 1967), p. 536.

he feels underlies the remaining two MM postulates. In stressing the difficulties of applying the MM concept to cost of capital considerations Durand provides an arbitrage model which, in essence, concludes that personal and corporate leverage are not equivalent, and therefore there is not sufficient arbitrage to support the MM proposition that market value is independent of capital structure.

The above controversy attracted a phalanx of thought and research which led to a prolific array of dissertations--many of which deal with relation between leverage and cost of capital.[1] One result of this outpouring was the independent but consensus finding that the cost of capital function is "saucer"-shaped (see Figure 8) as postulated in the traditional concept, and not horizontal as espoused by Modigliani and Miller.[2] These authors theorize that there is an optimal range of leverage for a given firm, in terms of its cost of capital. This range is indicated by the relatively flat segment of the firm's cost of capital curve which is controlled by the counter forces of tax benefits from debt and the

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[1] Mock, Financial-Decision Making, op. cit., p. 537.

[2] A representative cross section would include: Barges, op. cit., passim; Harold Peterson, "Risk and the Capital Structure of the Firm," Journal of Finance, 19 (March, 1964), pp. 120-121; and Ronald Frank Wippern, "Earnings Variability, Financial Structure, and the Value of the Firm," Journal of Finance, 19 (December, 1964), pp. 699-700.

increase in both the cost of capital and equity, with the result that the cost of capital changes very little.[1] Beyond this range, the marginal rate of interest demanded in the money markets exceeds the cost of capital.

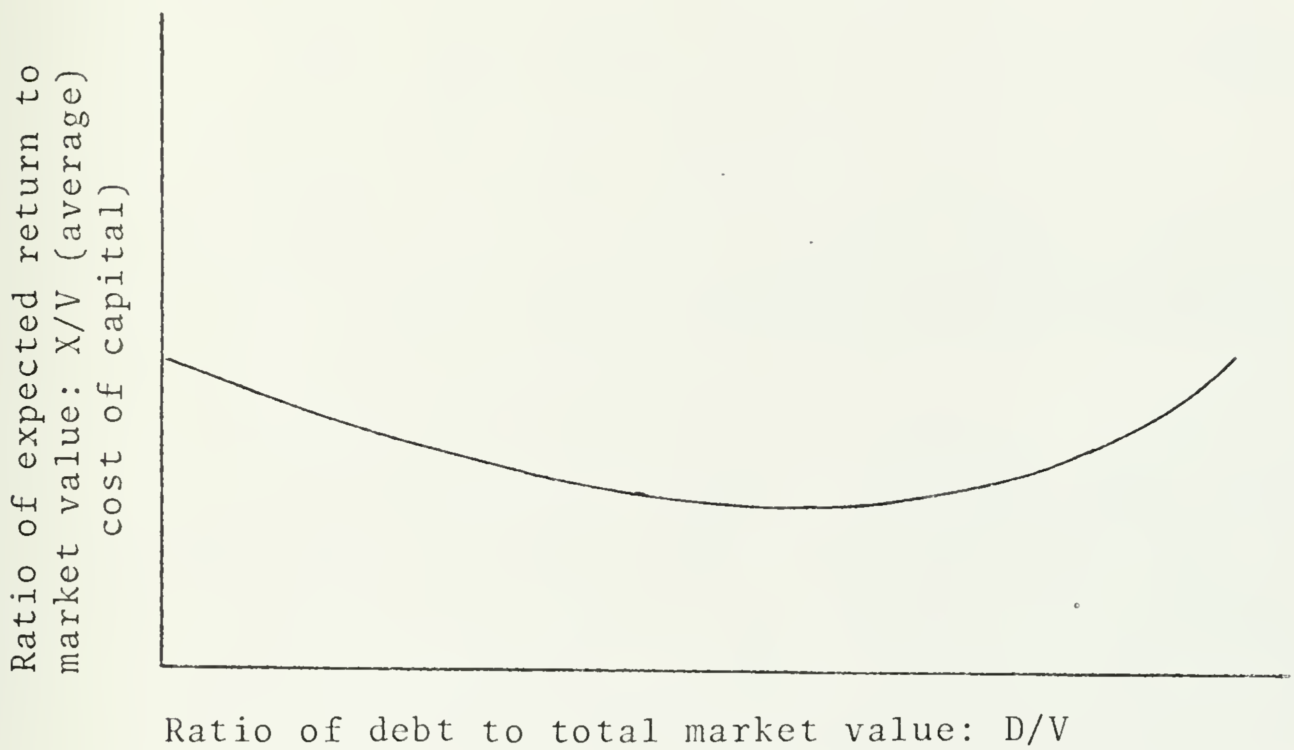

Figure 8. Average cost of capital as a function of debt to market value.

Source: Barges, *op. cit.*, p. 11.

X = The expected future profits before deduction of interest.

S = The market value of common shares of the firm.

D = The market value of debts of the firm.

V = The total market value of the firm (S+D).

X/V = The average cost of capital to the firm.

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[1]Mock, *op. cit.*, p. 537.

This writer sides with the "saucer" configuration and with Robichek and Myers, who disagree with the MM assumptions on the following bases:[1]

(1) Market imperfections may prevent full operation of the arbitrage process (that MM claim leads to the indifference to capital structure).

(2) Investors may (and obviously do) disagree on the risk class of any particular firm.

(3) Increasing leverage may face the firm to pass up profitable investments which an unlevered firm would undertake.

It is also this writer's view that with a substantial corporate tax on earnings, the use of debt, in moderation, can reduce the cost of capital to the firm; but, conversely, a firm forced to increase its debt-to-equity ratio beyond the optimal range established for its capital structure will soon find the interest exceeding its cost of capital. The relevance of this viewpoint and the preceding discussion is that while many firms rely on internally generated funds for capital undertaking, in theory a judicious mix of debt and equity will produce the optimal cost of capital. This, in turn, will maximize the investment opportunity spectrum of the firm and potentially

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[1]Alexander Robichek and Stewart C. Myers, *Optimal Financing Decisions* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965), pp. 40-41.

improve the residual owner's net present worth. If the manager's primary concern is the well-being of the firm's residual owners, this potential should be fully exploited.[1]

Specific Costs of Capital

If the economic theorists are in substantial disagreement as to the optimal decisions for capital administration and the composition of the investment base, there is little argument that the cost of capital to a firm at some point in time is the weighted average of the cost of equity and the cost of debt.[2] As implied in the foregoing discussion, businesses frequently employ more than one type of capital. Each of these sources will entail a cost. While explicit costs arise whenever the firm raises funds, implicit costs arise whenever funds are invested. The latter stems from the choice of alternatives, where choosing one investment means foregoing another, thereby incurring an "opportunity cost." Moreover, the measurement of a firm's cost of capital involves ambiguities, lively controversy, and forecasts subject to uncertainty.[3] Consequently, it is only after the cost of each type of capital has been determined that it is possible to calculate a meaningful composite cost of capital

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[1]Bogen, *op. cit.*, p. 17.48.

[2]J. R. Lindsay and Arnold W. Sametz, *Financial Management, An Analytical Approach* (Homewood, Ill.: Irwin, 1967), p. 321.

[3]Bogen, *op. cit.*, p. 17.50.

for a firm. The intent here is to incorporate a general concept of capital costs rather than pursue every possible avenue of funds source. Within this framework, only long-term debt will be addressed in the general term, borrowed capital. (However, the utilization and effect of short- and intermediate-term debt are acknowledged both in terms of financing and cost). Correspondingly, the treatment of stock will be confined to a limited situational analysis.

Long-Term Debt-Bonds[1]

As bond issues are the predominant means of raising long-term debt capital, the ensuing discussion will pertain to this financial instrument.

Debt capital entails the payment of interest, and incurs the liability of repayment of the face amount of the bond.[2] However, the cost of bonds will normally differ from the stated (nominal) interest cost.[3] The true cost of bonds is a function of their net proceeds, maturity value, number of years to maturity, and interest payments.[4]

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[1] As stated in the introductory paragraph, leasing will not be considered in this paper.

[2] In practice, the bonds may be retired by several methods, called back prematurely if so provided in the contract, or refinanced with a new issue.

[3] Bogen, *op. cit.*, p. 17.50.

[4] Mock, Schultz, Schultz and Shuckett, *op. cit.*, p. 208.

For example, with the 1970 prime rate hovering at 8-1/2 percent a highly rated 15-year bond with a face value of $1,000 and a stated rate of interest of 7 percent might sell at $950. For the use of the $950, the firm would pay $66.50 in interest and the $1,000 at the end of 15 years. (If the creditors stipulated a sinking fund retirement scheme, the issue would probably sell at an appreciably smaller interest rate). But the total cost of the bond to the firm is the interest plus a 1/15th share of the $50 discount, or a total cost of $69.83. Since the firm is presumably enjoying an accumulation of annual appreciation of principle, the average amount of funds involved is $950 (year 1) + 1,000 (year 20) divided by 2, or $975. This average available fund divided into its cost gives a percentage of 7.15 before taxes. Since interest expense is tax deductible the cost of the bond is:

$$7.15\% \times (1\text{-tax rate})$$

When the bond is sold at greater than face value (at a premium), the same general procedure is followed except that the average premium is deducted from the interest payment.

Normally a company will incur administration and/or handling expense in a bond issue. Whatever these costs, they are

applied against the bond proceeds to derive net proceeds per bond. In the above example, this would result in a figure less than $950, a lower average, and hence a higher cost of capital.

It should be noted that if earnings for the year (before taxes and interest) are negative, the tax shield does not apply and the cost of borrowed funds would be 7.15 percent.[1] However, for purposes of computing a cost of capital as a continuous concept, this refinement can be ignored.

Cost of Preferred Stock

Unlike debt, preferred equity is normally a "perpetual" security. While nonpayment of preferred dividends can precipitate undesirable consequences, no condition of default is created,[2] as is the case with debt equity. The explicit cost determination is relatively simple since normally the dividend rate is stated, payed into perpetuity, and not tax deductible.

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[1] Bogen, *op. cit.*, p. 17.52

[2] *Ibid.*, p. 17.53

While some issues carry a sinking fund provision, most do not. However, a "call" option is not uncommon and is desirable from the standpoint of management flexibility in adjusting the firm's capital structure. The calculation is simply:

$$\text{after-tax cost (\%)} = \frac{\text{stated dividend rate (\$)}}{\text{net proceeds per share (\$)}}$$

Cost of Common Stock

Whereas the cost of debt and preferred equity can be obtained via reasonably straightforward procedures, calculating the cost of common stock can be a variable process, rife with uncertainty.

As previously pointed out, estimation of future earnings is not an exact process, since earnings tend to be both unstable and uncertain. The cost problem is further complicated by the method of sale, which can be direct to the public, as additional shares to present stockholders under pre-emptive rights, or indirectly by way of convertible securities. Each of these procedures can affect common equity cost differently.[1] For the sake of brevity, and without significant sacrifice in comprehensiveness, only public sale of common stock will be quantitatively explored.

[1]Bogen, *op. cit.*, p. 17.53.

The initial consideration in determining the cost is to find the rate at which the market capitalizes the company's earnings, and to adjust this rate for the difference between the market price of the shares outstanding and the price per share (net proceeds) from the new issue. Again (at the risk of redundancy) future profits which are the basis of the capitalization rate determination cannot be predicted with certainty and will vary with time. Unless there is some significant indication that future profits will differ significantly from the past, it is customary to use past earnings as a basis for projecting profits.[1]

As will be illustrated in the example, diluation of earnings of existing residual owners resulting from the sale of stock is something to be avoided by management. This means that an investment should be accepted only when it is expected to return at least the same earnings per share as existed prior to the expansion.

For purposes of illustration, the following data are assumed in computing the cost of common equity.[2]

(1) 1,000,000 shares of common are outstanding;

(2) future average annual earnings are projected to be $5,000 after taxes.

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[1]Bogen, *op. cit.*, p. 17.54.

[2]This example is an adaptation of one in Bogen, *op. cit.*, pp. 17.55.

(3) Current market value is $50 per share.

(4) The estimated sale price of the stock will be $45 (to facilitate the sale).

(5) Underwriting cost, etc., will total $5 per share.

Evidently, the market capitalizes the shareholders' profits at 10% ($5,000,000 ÷ 1,000,000 shares x $50 per share). To avoid diluting the current stockholders' earnings, each new share must be accompanied by a $5 increase in earnings. (The administrative and operational lag experienced in most facility investments between investment and returns will be ignored).

Net proceeds per share are $45-5 = $40

.. the cost of this capital (earnings ÷ proceeds)=

$$\frac{\$\,5}{40} = 12.5\%$$

The difference between the 10% market rate of capitalization and this cost of capital to the company is the result of underpricing and flotation costs.

Cost of Internal Equity-Retained Earnings

The use of retained earnings to finance investments is becoming more widespread.[1] Consequently, cognizance of costs associated with the use of such funds would seem important. Yet according to some authorities, corporate officials frequently view these funds as without cost. If earnings are retained within

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[1] Bogen, *op. cit.*, p. 17.59.

the firm instead of being distributed to stockholders, these retained earnings represent a cost to the stockholders[1]--the residual owners of the firm. Use of these funds for any purpose should carry with it an implicit goal of improving the worth of the stockholder; otherwise, the stockholder could have taken these funds (as dividends) and used them for consumption or other investment opportunities. To quote Bierman and Smidt:

"The cost of using retained earnings is therefore the minimum yield that must be earned on additional investments within the company in order that the additional investment will be as valuable to the stockholder as a corresponding immediate increase in dividends."[2]

If this above philosophy is to be capsulized for cost of capital purposes, a valid label is "opportunity cost," and its computation should include the tax effect. Since dividends are taxable, opportunity cost can be expressed as an after-tax rate equal to the market rate of capitalization of the stock multiplied by (1-t). Where t is the stockholder's personal income tax rate.[3]

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[1]Bierman and Smidt, *op. cit.*, pp. 142-143.

[2]*Ibid.*, p. 144.

[3]Bogen, *op. cit.*, p. 17.60.

$\therefore$ Cost of retained earnings = $\frac{E(1-t)}{p}$

Where E = dollar return from reinvestment (of the dividend)

p = price of the stock

t = personal income tax rate

Obviously this method has a serious drawback. A "representative" tax rate must be selected that reflects in weight and degree the spectrum of rates of the individual stockholders. There have been a number of opinions offered as to the best means of computing this rate.[1]

This writer, at the risk of separating the corporation from its stockholders, considers internal equity equivalent to external equity, and in that light would exclude the tax consideration from the cost evaluation of retained earnings. By earning the same rate on retained earnings as on common stock, management will avoid diluting the owners' earnings. This is tantamount to considering the opportunity cost as that of the corporation instead of the stockholder. The dichotomy of dilution and tax bite are thus avoided and the basic contract remains in tact (i.e., the investor, owning his part of the

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[1]To mention two, Johnson (Financial Management) recommends a "marginal" tax rate; Hunt, Williams and Donaldson (Basic Business Finance) suggest "the minimum individual Federal tax rate."

corporation, holds management responsible for protecting his interests; and management, rather than the stockholder, decides how this is to be done). Under this premise, the problem remains of measuring cost such that it includes growth. Gordon and Shapiro have developed a model using dividends.[1]

$$ke = \frac{Do}{Mo} + br$$

where
- ke = cost of common equity capital after tax
- Do = current dividend per share
- Mo = current market price per share
- b = proportion of earnings retained and reinvested (a positive fraction less than 1)
- r = rate of return on the reinvestment of earnings

This formula is not a panacea as it involves a number of restrictive assumptions, which include constant growth rates in dividends and expected earnings.[2] However, in the eyes of the writer, it is easy to use and could be operationally modified to reflect historical trends. As for assumptions, when dealing with uncertainty, at least a majority of the esoteric aspects have to be frozen if the remainder are to be evaluated.

Weighted Cost of Capital

At this point, it might be well to update prior findings and point to the objective of the chapter. The major

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[1] M. J. Gordon and E. Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," *Management Science III*, (October, 1956), pp. 104-106.

[2] Mock, *op. cit.*, p. 218.

components of capital have been identified as debt, preferred and common equity, and retained earnings. These in turn have been analyzed as to concept and cost--all with purpose of deriving a single value that could be considered a weighted value or combined cost of capital. Once this has been derived, the way is open to the objective, that of applying a weighted cost of capital to the investment selection process.

According to Mock, once the cost of each type of capital has been determined, it is customary to combine them into a composite rate by weighing each class of capital by its share in the firm's capital structure. Thus, a firm with a capital structure of bonds and preferred and common equity could obtain a weighted cost of capital as follows:

Table 17

COMPUTATION OF A WEIGHTED COST OF CAPITAL

	Amount	x	After-Tax Rate	=	Dollar Cost
Bonds.	$ 15,000		2.5%		$ 375
Preferred Stock. . .	5,000		6.0		300
Common Equity. . . .	80,000		10.0		8,000
	$100,000				$8,675

Source: Mock, *op. cit.*, p. 219.

$$\text{The weighted cost of capital} = \frac{8,675}{100,000} = 8.675\%$$

In using this approach, the firm must answer two questions: (1) What weight is to be assigned each structural element? (2) Should book or market values be assigned the securities?[1] The answer to the first question depends on the firm. If it is striving for some target structure, and management believes it will attain it in the near future, the firm may use this as the basis for weighting the various fund sources. If it is content with the status quo, the present structure will probably be used. The second question is somewhat reminiscent of the accounting versus capital budgeting question aired at the beginning of Chapter III. Accounting entries (book values) are historical and have little to do with the current state of affairs. If stocks are sold, the current market determines theprice. Thus, from the standpoint of reality, market prices should be employed to weigh the costs of capital structural components.[2]

Significance and Application of Cost of Capital

While oversimplified, the investment opportunities confronting a firm can be visualized as a list of projects, arranged in order by descending rate of expected return. If the cost of capital, as developed in the foregoing discussion, is such that the weighted or composite figure is, say, 10 percent,

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[1] Mock, Schultz, Schultz and Schuckett, *op. cit.*, p. 220.

[2] *Ibid.*

then the only listed projects available to the firm are those with an estimated rate of return above 10 percent. If, on the other hand, the firm can rearrange its capital structure, and obtain an optimal mix of relatively low-cost debt and internal and external equity, the composite cost of capital will be minimized, at, say, 8 percent. In doing this, the availability portion of the investment list enlarges. This in turn provides management with a greater absolute number of investment possibilities, and also greater latitude in the acceptance of risk of original opportunities.

Without some idea of its cost of capital, management would be at a loss to establish a floor or hurdle with which to evaluate a given investment. An arbitrary cutoff rate could, if too high, lead to rejection of sound investments; and if too low, result in squandered resources.

What is implied in much of the authoritative writings on cost of capital is that in determining its cost of capital, a firm will, if not immediately, then eventually, seek its minimum cost of capital through an optimal mixture of debt and equity. This is a vast area of point/counterpoint, with neither full acceptance of concept nor procedure. However, for the purpose of this discussion, the writer will conclude that the rate

of return must surpass the cost of capital if capital expenditures are to be rational; that the merit of an investment should not be judged by the type of capital apparently raised to finance it (unless the capital structure of the firm is of one type), but rather by a weighted cost of capital; and that a minimum cost of capital is desired to enhance the flexibility of management in investment decisions.

CHAPTER VI

CONCLUSIONS

The goal of this study was to identify the quantitative factors that govern the selection of capital investments. Basic assumptions were that financing decisions had previously been made, would remain static, and that there was no leasing alternative. Other considerations such as management attitude and degree of organizational expertise were acknowledged as subjective influences, but as such were for the most part excluded from this study of quantitative determinants of capital investment selection.

The introductory chapter pointed out investment worth measurement as the heart of capital budgeting, and stressed the need for an adequate measurement tool. Chapter II presented an array of investment worth measurement techniques, emphasizing the underlying principles, and stressing the advantages and limitations of each method.

Chapter III gathered the ingredients essential to the application of the measurement techniques. The intent here was to utilize all pertinent conceptual considerations in constructing a selection procedure framework that would suffice for most classifications of investments.

Chapter IV was essentially a sequel of Chapter II in that the subject matter, risk, is an important factor in investment selection. However, it was felt that the special nature and pervasiveness of this factor warranted study in a separate chapter. Accordingly, representative quantifiable aspects were analyzed both as to significance and application. The need for a standard was strongly implied in the hypothetical application of investment selection procedures. This led to Chapter V where the cost of capital was examined as a common denominator against which to evaluate the results of the investment worth selection process.

<u>The following quantitative factors are concluded to be essential to the investment selection process</u>:

A. <u>A means of measuring investment worth.</u>

The technique should measure a proposed capital outlay, or series of outlays, in terms of effect on net earnings; wherein the amount of funds expected to be tied up over the estimated life of the project are properly related to the estimated investment earnings. It should utilize all information relevant to the decision on the investment proposal, should apply to all types of proposals, and allow for the realistic comparison of mutually exclusive and competitive projects. It should permit

indirect inclusion of quantitative risk calculations and be directly comparable to some standard for acceptance or rejection. Only the methods incorporating discounted cash-flows fulfill these requirements, and then only on a conditional basis.

B. <u>An analysis of cash outlays and inflows</u>.

The determination of an investment's cash outlays and income must be predicted with the greatest practicable accuracy for the period in which these cash-flows occur. In this way net earnings or outlays can be calculated for each period in the analysis time-frame. Once these values are obtained the nonassumptive requirements of the investment worth measurement technique are fulfilled.

The cash-flow method is based upon the concept that investment outlays are made now to acquire future cash inflows. By its very nature it implies a comparison of one thing to another. Since the value of a proposed investment depends on its future earnings, a correct estimate of earnings, and hence a correct cash-flow analysis must be measured by the total added earnings (or savings, or worth) from making the investment as opposed to not making it. As a corollary, a comparison of two proposed investments could be made by matching the periodic cash flows. They could also be compared on an absolute basis

(using the no-investment alternative as a common base).

The discounted cash-flow method is economically realistic in that it concerns only cash flows and disregards book values. It also relates the time pattern of investment outlays and resultant earnings to the inevitable diminishment of a given cash flow with time.

A. An analysis of risk (and uncertainty).

Risk can normally be quantified and often applied to various facets of the cash-flow analysis. Uncertainty is subjective and present to some degree in all quantitative estimates of future values. Certain types of investments are, by nature, composed of many variables of which little can be determined with any level of confidence. Some represent just another iteration of a process typified by consistent trends and few variables. In the first instance, risk may be the most potent determinant in the selection process; in the latter case, of least concern. Where it is recognized as a relevant factor in cash-flow projections, risk must be quantified and applied.

D. A standard for evaluation.

Investment entails the use of capital, and all types of capital have a cost to the firm. Most enterprises use more

than one type, which necessitates the calculation of a composite cost of capital. Except in rare instances, this cost, expressed as a percentage, represents the least a firm should realize as a rate of return on invested capital. In that regard, it also provides the interest rate with which to discount cash flows in the net-present-value method. While not to be considered a precise value, it is a warning that as a project return approaches this value, it should be subjected to closer scrutiny and increasing skepticism.

Having answered the research question, restatement of the four subsidiary questions, accompanied by comments based on respective material in preceding chapters, will conclude this paper.

What methodology is best suited to measurement of capital investment worth?

Two techniques of worth measurement with the discounted cash-flow concept as the underlying principle were found to be be theoretically correct and superior to all others. The net-present-value technique was found to have the following advantages:

(1) It is the easier of the two methods to use, since each term in the equation is essentially a simple fraction.

(2) When comparing mutually exclusive projects, it provides comparative values in monetary terms, avoiding the possibility of a higher percentage but dollar-foolish decision.

(3) In the uncommon instance of earnings becoming negative after the initial negative-positive transition, the NPV method will give but one correct answer. The yield method will give as many answers as there are sign changes--none of which are reliable.

(4) A profitability index can be compiled for the entire agenda of investment proposals.

The yield method, in providing a comparison on a rate-of-return basis, indicates the margin of clearance over the cost of capital hurdle, and hence an indication of the degree of uncertainty that can be tolerated. It also allows competitive projects to be arrayed by rate-of-return without a cost-of-capital standard. This might prove advantageous in a planning phase where the firm was undergoing a significant change in its capital structure, or where the proposed investments were expected to precipitate structure change. However, ultimately the yields obtained would have to be evaluated against some standard; i.e., the cost of capital.

On the basis of these findings, a firm would be prudent in using a synthesis of the NPV and yield methods.

Is there any significance in a reinvestment assumption in the net-present-value and yield methods?

The NPV technique uses the cost of capital as the rate of interest. Therefore, projects ranked by this method will not be affected by a change in the reinvestment assumption. The yield method implies that returns from the initial investment can be reinvested at the same rate as that of the original investment. Consequently, a significant change in reinvestment conditions could result in a reshuffling of projects ranked by the yield method. Reinvestment is implied in both methods, but only for the yield method does this assumption loom significant.

What conceptual considerations comprise an investment cash-flow analysis?

A. Absolute and relative returns.

A project can have both. The absolute return comes from the net cash flow of the project revenues and operating costs. The relative return is based upon the difference between the net cash flows of the project and that of an alternative. The former may often be unobtainable, but the latter can be derived anytime there is an alternative. Consequently, there

can be as many relative returns as there are alternative projects. Relative returns are blind to proper timing and the quality of the alternative. Absolute returns tend to ignore interaction effects which may be crucial to the selection decision. A thorough analysis should incorporate both, assuming an absolute rate can be obtained.

B. Investment timing.

An investment opportunity today will likely not remain static as to its variable elements; and hence tomorrow's decision may address an opportunity of different magnitude. Delay can be very expensive in terms of available costs or foregone revenues. Conversely, a premature replacement investment may be obsolete before it generates its first revenue. Few investments warrant implementation without considerable study of the "WHEN" aspects. Timing may prove to be the aspect that determines the ultimate success or failure of the investment.

C. Taxation.

Taxes are as much a part of business decisions as the facets of business that they affect. Whether it be projected earnings, choice of depreciation method, or timing, the cash-flow analysis of an investment is linked to the tax

rate. It is not surprising that investment decisions are commonly influenced by the tax aspect. Since the underlying worth of a project is more fully revealed when tax effects are imparted to the analysis, a pre-tax list of project rankings might undergo considerable rearrangement when tax implications are incorporated into the equation.

D. Accounting.

The financial accounting conventions indispensible to financial reporting are not appropriate for investment analysis. The former are related to net income for an accounting period, and book values, while the cash-flow concept is concerned with the magnitude and timing of future cash outlays and inflows. Furthermore, investment outlays properly budgeted as capital expenditures are not consistent with the accounting distinction between capitalized and expensed outlays.

How should risk and uncertainty be applied to the investment selection process?

Risk, where it can be identified and quantified, should be interjected as a probability factor in the on-going evaluative process from where it appeared. The problem is identification: the burden is quantification; but application is essentially mechanical.

The identification problem is primarily one of administrative technique and personnel education, including awareness of such aspects as the influence of risk upon the cost of money. The onerous but essential burden of calculation can be reduced to acceptable levels by computer programming. The methodology is well-documented, and considerable effort is being applied to the development of more sophisticated techniques. However, utilization is already far behind research.

Where uncertainty looms, and economic evaluation and mathematical probabilities are not available or have been exhausted, quantitative analysis stops and judgment continues alone.

What is the relationship among investment selection, cost of capital and capital structure?

This question was overly ambitious in that by itself it would provide a reasearch requirement of thesis magnitude. The basic research question discussed cost of capital and investment selection in a context which, while parochial, defined a relationship of the former posing a standard for the latter. The relationship between capital structure and cost of capital was not established beyond the tenuous observation that inclusion of low-cost debt capital will, under certain capital structures in certain firms, lower the cost of the firm's capital, but that increased use of debt capital results in higher costs, reflecting risk consciousness on the part of

lenders. It is therefore implied that an optimal cost of capital, which would enhance investment selection from a range-of-opportunity standpoint, could be obtained from a proper mix of debt and equity capital. However, no attempt was made to ascertain the existence of a model defining such a relationship.

BIBLIOGRAPHY

Books

Anthony, Robert N. Management Accounting. Homewood, Ill.: Richard D. Irwin, Inc., 1970.

Barges, Alexander. The Effect of Capital Structure on the Cost of Capital. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963.

Baumol, William J. Economic Theory and Operations Analysis. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965

Bierman, Harold, Jr., Bonini, Charles P., Fouraker, Lawrence F., and Jaedicke, Robert K. Quantitative Analysis for Business Decisions. Homewood, Ill.: Richard D. Irwin, Inc., 1965.

Bierman, Harold, Jr., and Smidt, Seymour. The Capital Budgeting Decision. New York: The McMillan Company, 1960.

Bogen, Jules I. Financial Handbook. New York: The Ronald Press Company, 1964.

Childs, John F. Long-Term Financing. Englewood Cliffs, N.J.:

Dean, Joel. Managerial Economics. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1951.

____________. Capital Budgeting. New York: Columbia University Press, 1951.

Horngren, Charles T. Accounting for Management Control: An Introduction. Englewood Cliffs, N.J.; Prentice-Hall, Inc., 1965.

Hunt, Pearson, Williams, Charles M., Donaldson, Gordon. Basic Business Finance. Homewood, Ill.: Richard D. Irwin, Inc., 1966.

Johnson, Robert W. Financial Management. Boston, Mass.: Allyn and Bacon, Inc., 1962.

Lindsay, J. R. and Sometz, Arnold W. Financial Management. Homewood, Ill.: Richard D. Irwin, Inc., 1967.

Mao, James C. T. Quantitative Analysis of Financial Decisions. London: Collier-Macmillan, Ltd., 1969.

Merrett, A. J. and Sykes, Allen. The Finance and Analysis of Capital Projects. New York: John Wiley & Sons, Inc., 1963.

Mock, Edward J. Financial Decision-Making. Scranton, Pa.: International Textbook Co., 1967.

Mock, Edward J., Schutz, Robert E., Schultz, Raymond G. and Shuckett, Donald Hart. Basic Financial Management. Scranton, Pa.: International Textbook Co., 1968.

Murdick, Robert G. and Deming, Donald D. The Management of Capital Expenditures. New York: McGraw-Hill Book Co., 1968.

Niswonger, C. Rollings and Tess, Philip E. Accounting Principles. Cincinnati, Ohio: South-Western Publishing Co., 1965.

Porterfield, James T. S. Investment Decisions and Capital Costs. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965.

Robichek, Alexander and Myers, Stewart C. Optimal Financing Decision. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965.

Shubin, John A. Managerial and Industrial Economics. New York: The Ronald Process Co., 1961.

Spencer, Milton H and Siegelman, Louis. Managerial Economics - Decision-Making and Forward Planning. Homewood, Ill.: Richard D. Irwin, Inc., 1959.

Solomon, Ezra. The Management of Corporate Capital. Glencoe, Ill.: The Free Press of Glencoe, Ill., 1959.

Staudt, Thomas A. and Taylor, Donald A. A Managerial Introduction to Marketing. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970.

Terborgh, George. Business Investment Management. Washington, D.C.: Machinery and Allied Products Institute, 1967.

Wasson, Chester R. The Economics of Managerial Decision. New York: Appleton-Century-Crafts, 1965.

Weston, J. Fred and Brigham, Eugene F. Essentials of Managerial Finance. New York: Holt, Rinehart and Winston, Inc., 1968.

Articles

Baldwin, Robert H. "How to Assess Investment Proposals," Harvard Business Review, XXXVII, No. 3 (May-June, 1959), pp. 98-104.

Dean, Joel. "Measuring the Productivity of Capital," Harvard Business Review, XXIII, No. 1 (January-February, 1954), pp. 120-129

Durand, David. "The Cost of Capital in an Imperfect Market: A Reply to Modigliani and Miller," American Economic Review, XLIV (June, 1959), pp. 639-655.

Gordon, M. J. and Shapiro E. "Capital Equipment Analysis: The Required Rate of Profit," Management Science, III, No. 3 (October, 1956), pp. 102-110.

Hertz, David B. "Risk Analysis in Capital Investment," Harvard Business Review, XLII, No. 1 (January-February, 1964), pp. 95-106

Hirshleifer, J. "On the Theory of Optimal Investment Decision," Journal of Political Economy, LXVI, No. 4 (June, 1958), pp. 329-352.

Magee, John F. "How to Use Decision Trees in Capital Investment," Financial Decision Making, ed. Edward J. Mock, Scranton, Pa.: International Textbook Company, 1967.

______________. "Decision Trees for Decision Making," Harvard Business Review, XLII, No. 4 (July-August, 1964), pp. 126-138.

Margo, B. A. "Latest Word in Profitability," Factory, January, 1958, pp. 60-63.

Modigliani, Franco and Miller, Merton H. "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, XLIV, No. 3 (June, 1958), pp. 261-297

Peterson, Harold, "Risk and the Capital Structure of the Firm," Journal of Finance, XIX (March, 1964), pp. 120-121.

Pollach, Gerald A. "The Capital Budgeting Controversy: Present Value vs. Discounted Cash Flow Method," National Association of Accountants Bulletin, (November, 1961), pp. 1-26.

Ruel, Raymond I. "Newest Way to Figure Payoff," Factory, October, 1955, pp. 92-96.

____________. "Profitability Index for Investments," Harvard Business Review, XXXV, No. 4 (July-August, 1957), pp. 116-132.

Weaver, J. B. and Reilly, R. J. "Interest Rate of Return for Capital Expenditures Evaluation," Chemical Engineering Progress, October, 1956, pp. 406-410.

Wippern, Donald Frank. "Earnings Variability, Financial Structure, and the Value of the Firm," Journal of Finance, XIX (December, 1964), pp. 699-700.

Woods, Donald H. "Improving Estimates that Involve Uncertainty," Harvard Business Review, XLIV, No. 4 (July-August, 1966), pp. 91-98.

Reports

National Association of Accountants. Return on Capital as a Guide to Managerial Decisions. Research Report No. 35. New York: National Association of Accountants, 1959.

National Industrial Conference Board. Controlling Capital Expenditures. Report No. 62. Prepared by the Capital Appropriations Committee. New York: National Industrial Conference Board, 1963.

Unpublished Material

Oscarson, Edward A. "Capital Investment Analysis in the Navy." Master's thesis, George Washington University, 1967.

Ruel, Raymond I. "Calculating the Return on Proposed Projects, Equuipment or Plant Facilities, "Industrial Educational Institute, Boston, Mass., 1956 (Mimeographed.)